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ABSTRACT

Tacit Collusion in Repeated Auctions*

by Andreas Blume and Paul Heidhues

We study *tacit collusion* in repeated auctions in which bidders can only observe past winners and not their bids. We adopt a stringent interpretation of tacit collusion as *collusion without communication about strategies* that we model as a symmetry restriction on repeated game strategies: Strategies cannot discriminate among initially nameless bidders until they have become named through winning an auction. We obtain three classes of results: (1) Completely refraining from using names, i.e. strengthening the symmetry constraint, rules out collusion altogether, and even if naming is permitted, as per our definition of tacit collusion, the lack of communication limits collusive strategies and payoffs among impatient bidders. (2) If communication is allowed, there are sustained improvements over bid rotation and competitive bidding among patient bidders. (3) These gains extend to tacit collusion among patient bidders. However, whether tacit or not, collusion need not be efficient.

Keywords: tacit collusion, auctions, supergames, strategic uncertainty, language, attainability

JEL Classification: C73, D44

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ZUSAMMENFASSUNG

Stillschweigende Kollusion in wiederholten Auktionen

Der Beitrag untersucht die Möglichkeiten von „tacit collusion“ (stillschweigender Kollusion) in wiederholten Auktionsspielen in welchen nur die vergangenen Gewinner nicht aber deren Gebote bekannt sind. Dabei wird „tacit collusion“ als kollusives Verhalten ohne Absprachen zwischen den Bietern interpretiert. In dem Artikel werden insbesondere auch vor dem Spiel getroffene Absprachen über Strategien ausgeschlossen. Das Fehlen solcher Absprachen wird durch Symmetrierestriktionen modelliert: Strategien können solange nicht zwischen anfangs „namenlosen“ Bietern unterscheiden, bis diese sich durch das Gewinnen einer Auktion von den anderen Bietern differenzieren. Es werden drei Arten von Ergebnissen hergeleitet: (1) Stärkt man die Symmetrierestriktionen und verlangt symmetrisches Verhalten in jeder Periode, so kann keine Kollusion auftretten. Aber auch weniger starke Symmetrierestriktionen, die prinzipiell eine endogene Rollenverteilung ermöglichen, schränken die möglichen Kollusionsgewinne bei ungeduldigen Bietern ein. (2) Erlaubt man vor dem Spiel getroffene Absprachen über die Strategiewahl, so können hinreichend geduldige Bieter unbegrenzt höhere Gewinne erhalten als die Gewinnne bei wiederholtem Konkurrenzverhalten oder bei einer einfachen Bieterrotation. (3) Dies gilt auch für Kollusion ohne Absprachen falls die Bieter hinreichend geduldig sind. Jedoch, ob mit oder ohne a-priori Absprachen, effiziente Kollusion kann selbst bei extrem geduldigen Bietern unmöglich sein.

1 Introduction

Despite an extensive literature on "tacit collusion" in economics, there is considerable ambiguity about the use of the term. Often, as e.g. in Tirole [1988], it is identified with collusion "in a purely noncooperative manner," thus allowing for explicit coordination on how to interact in the game and making a large variety of behaviors supported by repeated interaction eligible. Others, such as Carlton and Perloff [1999], emphasize the absence of explicit communication as defining tacit collusion. The latter definition is in line with legal practice in the US, where any explicit communication between competitors coordinating behavior in the market place is considered illegal.[1] In the absence of explicit communication on how to play the game, competitors face considerable *strategic uncertainty.* With reference to the FCC auctions, this leads Cramton [1997] to state that: *"Fortunately, tacit collusion is easily upset. It requires that all the bidders reach an implicit agreement about who should get what. With thirty diverse bidders unable to communicate about strategy except through their bids, forming such unanimous agreement is difficult at best."* Yet, in the standard noncooperative analysis of tacit collusion such strategic uncertainty plays no role. In practice, players can only overcome the strategic uncertainty by observing each others behavior in the auction (respectively market place).

But how should one model bidders' inability to communicate outside of the trading mechanism itself? This question is particularly interesting in the auction setting because, unlike in repeated oligopoly models, there are no simple symmetric rules to sustain cooperation like a trigger strategy where *all* firms charge the monopoly price until a deviation is observed in which event *all* firms revert to competitive pricing for a specified number of periods. Instead, as emphasized in the above quote by Cramton, an important question is "who should get what?" One can easily imagine that the inability to communicate makes it harder to ascribe role differences to different players and hence makes it harder to answer this question. Yet, despite the intuitive plausibility, we are not aware of any paper that formally addresses these important issues.

To model the strategic uncertainty created by the lack of communication outside of the

[1] See Ayres [1987] who writes that "[l]egal scholars have traditionally distinguished between explicit and tacit collusion. The law punishes the former, so that the act of communication is of central importance. For economists, however, this distinction has no meaning."

institution itself, we use symmetry constraints. Suppose, for example, bidders would like to use a bid-rotation scheme and that such a bid rotation can be supported by a noncooperative equilibrium. In such an equilibrium, one bidder is designated to win in the first period. In the absence of communication between bidders, this raises the question how this bidder is selected. In a symmetric auction environment there is nothing to single out a particular bidder as the natural winner. In other words, bidders face strategic uncertainty, which requires that all bidders use symmetric bidding behavior in the first period. On the other hand, bidders can acquire different roles over time through the repeated interaction in the auction itself.

For the most part, we focus on *attainable strategies.* Underlying the concept of attainable strategies is the idea that only when a player's observed past behavior differentiates him from his rivals, beliefs and future actions can be tied to this behavior. Intuitively, one can think of the player as obtaining a name that allows him to play a different role then his rivals in the future. So after the first period, if the winner is observed, he can be required to play a different role in the continuation equilibrium from all of his rivals. If the observable histories of his rivals are identical, however, they are required to use symmetric strategies in the continuation game. Similarly, if another player wins, he obtains a name and can be treated differently in the future. In this equilibrium concept, the history of the game allows bidders to construct substitutes for natural language in the interest of collusion.

We address the issue of communication and strategic uncertainty in a simple infinitely repeated auction setting (including first-price, second-price, and all-pay auctions) in which players' values are drawn independently every period and only the winner is announced.[2] In this setting, we investigate the constraints placed by attainability on collusive equilibrium strategies and payoffs. We examine both improvements over bid rotation and over competitive bidding.[3]

We proceed in three steps. First, we show that naming is essential. For example, strategies that never rely on names do not improve on competitive bidding in standard auctions. Second, ignoring the attainability constraint for the moment, we construct a collusive equi-

[2]This auction enviroment was recently analyzed in Skrzypacz and Hopenhayn [1999]. See Section 6 on how our work relates to theirs.

[3]Bid rotation is a natural reference point because McAfee and McMillan [1992] show that in the static case without side payments, bid rotation is the optimal form of collusion under weak distributional assumptions.

librium with the property that the difference between the payoff of this collusive equilibrium and the payoff from bid rotation (or competitive bidding) has no upper bound if players can be made sufficiently patient. Third, we construct an attainable equilibrium that relies on three phases: a naming phase, a reward phase, and a collusive phase. The naming phase serves to remove the attainability restrictions, the reward phase helps to provide appropriate incentives for the naming phase, and the collusive phase implements the collusive equilibrium constructed in the second step. This equilibrium payoff dominates bid rotation (and competitive bidding) as well because the length of naming and reward phases is bounded, whereas the payoff difference during the collusive phase is not.

Our main result is that as the discount factor approaches one, the inability to communicate about strategies does not restrict the average payoff that players can obtain and hence does not restrict their ability to collude. We also prove that for sufficiently patient bidders bid rotation is never the optimal form of tacit collusion in any standard auction and that players can obtain a higher payoff than in the best static equilibrium in first- and second-price sealed-bid auctions.

In establishing our findings, we considerably strengthen existing results on noncooperative equilibria in repeated auctions. Among other things, we prove a novel *anti-folk theorem:* In any perfect public equilibrium of the repeated second-price sealed-bid auction with more than two bidders, the average per-period payoffs are bounded away from the full collusive gain.

The rest of the paper is organized as follows. In Section 2, we introduce the basic model and formalize language constraints such as attainability. In Section 3, we show that restrictions on the use of bidders' names constrain payoffs from collusion. In Section 4, we construct equilibria with unbounded gains over bid rotation and competitive bidding. We then use these results in Section 5 to construct attainable equilibria that beat competitive bidding and bid rotation. In Section 6, we discuss the related literature. Section 7 concludes.

2 Setup and Language Constraints

In this section, we describe the auction environment, discuss the solution concept, and formally define attainability.

There are $N \geq 2$ bidders participating in an infinite sequence of auctions for separate objects. The valuation of bidder i for an object in period t is denoted by $v_i(t)$. We assume that these valuations are drawn independently across players and time, from a fixed distribution with *c.d.f.* $F(\cdot)$. Let F have a density function $f(v)$ that is strictly positive over the interval $[v^l, v^h]$ with $v^l < v^h$ and $v^h > 0$. All players are risk neutral and have common discount factor δ. We assume that players have access to a public randomizing device. This structure and the auction rules are common knowledge.

Following Skrzypacz and Hopenhayn [1999], henceforth SH, we restrict attention to *standard auctions* defined as follows:

Definition 1 *A* standard auction *is any set of auction rules that satisfy the following conditions:*

1. *A buyer can make any nonnegative bid.*
2. *The buyer who submits the highest bid is awarded the good.*
3. *The expected payment from a zero bid is zero.*
4. *Auction rules are anonymous, so that each player is treated symmetrically.*
5. *There exists a unique common equilibrium bidding strategy, $b^e(v)$, which is strictly increasing in v for $v \geq 0$ and with zero expected payoff for a buyer with value zero.*

Throughout, we will consider the case of $v^l = 0$; the more general formulation was only needed to extend properties of standard auctions to environments with induced valuations that are translates of the original valuations. Such translations will play an important role because they arise naturally when adding continuation payoffs in the repeated game to current valuations.

Examples of standard auctions are first-price auctions, second-price auctions, and the all-pay auction.[4] For future reference, note that incentive compatibility implies that in equi-

[4] The class of auctions also includes the third-price auction, which was used in Kagel and Levine, and higher-price auctions if there exists a sufficient number of bidders. It also includes auctions in which the highest bidder obtains the good and pays a price equal to the weighted average between the first and the second highest bid, which were analyzed in Plum [1992].

librium a bidder's expected payment is nondecreasing in his own bid and that the maximum expected payment is bounded by the highest possible value.

At time t buyers know their own valuation of the good currently being sold but not of the goods that are being sold in the future or the valuations of other buyers. It is common knowledge that participation in the auction is limited to N players. We assume that the winner of the object is publicly observable but not the winning bid.

Let $H(t)$ denote the history of the game up to, but not including, time t. This history contains information about the bids of the players, the realizations of the public randomization device, and the history of the winners in all auctions up to time t. Denote by $h(t)$ the public history of the game up to time t, that is the identities of the winners and the realizations of the public randomization device in the corresponding auctions.[5]

We study Perfect Public Equilibria (PPE), as defined in Fudenberg, Levine and Maskin [1994]. In a PPE, players' strategies only depend on the publicly available history, and strategies form an equilibrium after every history. Deviations to nonpublic strategies are permitted but irrelevant. As long as the other players use public strategies, private history can be ignored when evaluating future payoffs. PPEa are recursive and therefore dynamic programming techniques apply: The one-deviation principle holds, i.e. one can verify that a particular strategy profile is a PPE by checking that no player can gain after any history by deviating from his prescribed strategy once and conforming with it forever after.

Any PPE can be made into a Perfect Bayesian Equilibrium (PBE) by choosing *any* beliefs that conform with Bayes' rule. Since behavior depends only on public history, these beliefs do not matter for evaluating future payoffs. PPEa satisfy one of the two conditions for a sequential equilibrium, sequential rationality. Since those beliefs that are not tied down by Bayes' rule are arbitrary, the other condition, consistency, is immaterial. Thus, we may think of PPEa as sequential equilibria.

If bidders are unable to communicate, they may find it more difficult to overcome *strategic uncertainty.* While we still lack a good formal representation of strategic uncertainty, we

[5]Strictly speaking, the public history should be defined as the largest set of common knowledge events. In some special standard auction environments this is larger than $h(t)$. For example, in the second-price sealed-bid auction with two bidders, the second price is common knowledge. For our positive results, which establish the existence of desirable PPE, restricting the public history is without loss of generality. Note also that when proving our anti-folk theorem for second-price sealed-bid auctions, we consider the case of more than three bidders in which case $h(t)$ is the largest set of common knowledge events.

can at least investigate some necessary restrictions on equilibria (provided we retain an equilibrium perspective). We use symmetry to capture some of the implied restrictions. In particular, we believe that *ex ante*-identical bidders must have identical beliefs. Then, adopting the perspective that a player's strategy is an expression of the beliefs of the other players about his behavior (see for example Aumann and Brandenburger [1995]), identical bidders must use identical strategies.

An advantage of this approach is that we can study to what extent inability to directly communicate hinders collusion. We note that symmetry restrictions can be weakened through repeated interaction and one can think of a variety of different symmetry conditions, reflecting some structure that may for example have arisen out of prior interaction among the bidders.

Our central symmetry condition on strategy profiles is *attainability.* A strategy profile is *attainable* if at any point in time it respects the remaining symmetries in the game; e.g., if all bidders are initially symmetric, they all have to use the same first-period bid function. They may use different bid functions in later periods to the extent that they are distinguished by the history of wins.

This condition can be formalized as follows: Denote the bid function of player i in period t by $b_i(v_i(t), h(t), \sigma)$, where σ denotes a strategy profile. Let $\phi : N \to N$ be a permutation of players (i.e. a one-to-one function on the set of players). Recall that the public history $h(t)$ is a sequence containing the identities of all winners and the realizations of the public randomization device up to date t, that is $h(t) = \{i_\tau, r_\tau\}_{\tau=0}^{t-1}$ (we can create a fictitious 0-player for those instances where nobody wins). The permutation ϕ acts in the obvious way on the history $h(t)$, i.e. we can define (with slight abuse of notation) $\phi(h(t)) := \{\phi(i_\tau), r_\tau\}_{\tau=0}^{t-1}$.

Definition 2 *A strategy profile is* attainable *if*

$$b_i(v, h, \sigma) = b_{\phi(i)}(v, \phi(h), \sigma), \ \forall i, \forall \phi \text{ and } \forall h.$$

Under an attainable strategy, a player who won the object in a given period may be perceived and treated differently from other players thereafter. Intuitively, through winning the player obtains a name, and beliefs and actions can be tied to that name.

To emphasize the role of such names, we will sometimes consider even more stringent restrictions on the reliance on player identities. We will consider strategies that *never* depend on names. Such *name-free* strategies can only depend on the binary sequence of wins and "no wins" and not on the associated names of winners. Let $\rho : N \cup \{0\} \to \{0, 1\}$ be a function that maps 0 into 0 and all elements of N into 1 and define $\rho(h(t))$ in the obvious way. When applied to a history h, this function eliminates all the information on the identity of winners, while retaining the information on whether there was a win at all in a given period.

Definition 3 *A strategy profile is* name free *if*

$$b_i(v, h, \sigma) = b_{\rho(i)}(v, \rho(h), \sigma), \ \forall i \text{ and } \forall h.$$

Both attainable and name-free strategies are "symmetric" in the sense of not depending on an *a priori* assignment of names. They differ in that attainable strategies permit elaborate naming schemes whereas name-free strategies permit no naming at all. Inbetween are strategies that permit naming but rule out elaborate naming schemes. We are particularly interested in strategies that make only minimal use of names. *Name-simple* strategies only recall the name of the last winner; they do not vary with permutations of other previous winners. Let ϕ_i denote a permutation that fixes (the name of) bidder i, and $i(h)$ the last winner in history h.

Definition 4 *A strategy profile is* name simple *if it is attainable and*

$$b_i(v, h, \sigma) = b_i(v, \phi_{i(h)}(h), \sigma), \ \forall i, \forall \phi \text{ and } \forall h.$$

Finally, it is worth mentioning that attainability can be modulated to account for numerous preexisting structures on the set of bidders. Let G be any set of permutations of the set of bidders N; in particular, G can be a strict subset of the set of all permutations of N. Then we can define a version of attainability that respects only the symmetries embedded in G.[6]

Definition 5 *A strategy profile is* G-attainable *if*

$$b_i(v, h, \sigma) = b_{\phi(i)}(v, \phi(h), \sigma), \ \forall i, \forall \phi \in G \text{ and } \forall h.$$

[6] Blume [2000] argues that for such a set to be common knowledge among the players, it has to form a group.

For a simple example where G-attainability would be appropriate, consider three firms who repeatedly bid on a sequence of public contracts and who may wish to arrange a bid rotation. If they could talk to each other, ignoring incentive issues, they could easily agree on who the first contract should be awarded to. Without communication they can try to rely on what they know about each other and what they learn through the course of a repeated interaction. Denoting the three firms by generic labels $*, \#$, and $\&$, suppose it is commonly known among them that their order by market share (from largest to smallest) is $*, \#, \&$, the order by profitability is $\#, \&, *$, and the order by number of employees is $\&, *, \#$. Each of these orders could be used as a focal point to generate a bid-rotation.

Considering the first period, the firms suffer from an embarrassment of riches: with three possible orderings it may not be obvious which one to use. Thus, initially, all firms may be forced to behave identically. However, the common knowledge of the set of three orderings, while useless initially, proves useful in the second round. If, for example, the first contract is awarded to firm $\#$, then *all* three orderings suggest that the next contract be awarded to firm $\&$ (and similarly if one of the other two firms is awarded the first contract).[7]

3 Language Constraints in Standard Auctions

In this section, we give examples in which language constraints limit optimal collusion. Evidently, language constraints rule out certain types of collusive behaviors. For example, bid rotation schemes that start at the beginning of the game are not attainable. More importantly, sufficiently strong language constraints may rule out otherwise feasible collusive equilibrium *payoffs.*

We begin by looking at name-free equilibria in repeated standard auctions. In other settings name-freeness need not rule out efficient payoffs; e.g. trigger strategies in the infinitely repeated Prisoners' Dilemma are name-free. In the present setting however, name-freeness entirely rules out collusion in repeated standard auctions.

In the following, we say that a player *bids competitively* in period t if the bid function

[7]More formally, the three orderings can be identified with a set of three permutations which together form a cyclic subgroup G of the group of all permutations of a set of three elements. The fact that all firms are named once a single firm is named is reflected in the stabilizer of each generic label being the group identity.

he uses in period t is identical to the unique symmetric equilibrium bid function of the one-shot auction. For any public history h, use (h, w) to denote the history h followed by a win and (h, n) the history h followed by no win. Let $t(h)$ be the time period immediately following history h. For any history h and name-free strategy σ, let $V(h; \sigma)$ be the (common) continuation payoff from σ after history h. If σ prescribes behavior other than competitive bidding after history h, we call h a noncompetitive history.

Lemma 1 *In any infinitely repeated standard auction, there exists no name-free perfect public equilibrium with higher expected payoffs than from repeated competitive bidding.*

Proof: Note that contingent on bidder i submitting a bid, the bid b_i does not affect the public history if strategies are name-free. The only incentives of bidder i that possibly involve variations in continuation values are between bidding and not bidding. Clearly, if a bidder with value v prefers to bid, any bidder with value $v' > v$ will also bid. Thus, for any history h there will be a critical current period valuation $\eta(h) \geq 0$ in period $t(h)$ below which no one will bid and above which everyone will bid.

If the history h is noncompetitive with critical type $\eta(h)$, then the repeated-game payoff to a bidder with value v from not bidding in period $t(h)$ is

$$0 + \delta((F(\eta(h)))^{N-1}V((h, n); \sigma) + (1 - (F(\eta(h)))^{N-1})V((h, w); \sigma)),$$

whereas the payoff from submitting a zero bid equals

$$v(F(\eta(h)))^{N-1} + \delta V((h, w); \sigma).$$

Hence, any $\eta(h) \in (0, v^h)$ satisfies

$$\eta(h) = \delta(V((h, n); \sigma) - V((h, w); \sigma)).$$

Thus if $\eta(h) \in (0, v^h)$ then it is equal to the discounted differences in the continuation values.

Since a bidder with value $v > \eta(h)$ does not alter history by varying his bid b, he chooses b to maximize payoffs in the one-shot game where bidders with value less than $\eta(h)$ do not submit a bid. By assumption, for any $\eta(h)$ there exists a unique symmetric bid function

$b(v,h,\sigma)$, which is strictly monotone increasing for all $v > \eta(h)$. Since the highest bidder is awarded the good, incentive compatibility requires that the expected current period payoff for a bidder with value v is equal to

$$\eta(h)F^{N-1}(\eta(h)) + \int_{\eta(h)}^{v} F^{N-1}(t)dt.$$

Note that

$$\eta(h)F^{N-1}(\eta(h)) + \int_{\eta(h)}^{v} F^{N-1}(t)dt - \int_{0}^{v} F^{N-1}(t)dt < \eta(h), \quad \forall\ \eta(h) > 0.$$

Therefore, conditional on $\eta(h) > 0$, the current period gain in $t(h)$ over competitive bidding is less than the discounted difference in continuation values from no one winning versus someone winning. Let period $\hat{t}$ be the first period in which the strategy σ does not prescribe competitive bidding on the equilibrium path (i.e. in which $\eta(h) > 0$). Replace strategy σ by a strategy σ' that prescribes competitive bidding in period $\hat{t}$ and that has continuation value $V((h(\hat{t}), n); \sigma)$ independent of the prior history. The new strategy has strictly higher expected payoff. In the same manner, every noncompetitive period can be replaced by a period of competitive bidding, each time strictly increasing expected payoffs. Evidently, the payoffs from the strategies generated from these successive improvements converge to the payoffs from repeated competitive bidding. □

Indeed, name-freeness completely determines the bidders' equilibrium behavior in every period.

Proposition 1 *In any infinitely repeated standard auction, there is a unique name-free perfect public equilibrium. In this equilibrium, bidders bid competitively after every history.*

Proof: We observed earlier that after any history h there exists a critical value $\eta(h)$ below which players refrain from submitting a bid and above which all players bid. If $\eta(h) = 0$ for all h then players use the competitive bid function $b^c(v)$ after any history and thus Proposition 1 holds.

Suppose $\eta(h') > 0$ after some history h'. Then we claim that there exists a profitable deviation for bidder i. Consider the following bid function $\tilde{b}(v,h,\sigma)$: bid $b(v,h,\sigma)$ for any

$v \in [\eta(h), v^h]$ and 0 for all $[0, \eta(h))$. In any period in which $\eta(h) = 0$ the payoff of this bid function is identical to the payoff of competitive bidding. In any period in which $\eta(h) > 0$ the expected contemporaneous payoff from $\tilde{b}(v, h, \sigma)$ must be greater than the expected contemporaneous payoff from $b(v, h, \sigma)$.

Thus after history h' there exist a strategy according to which bidder i bids $\tilde{b}(v, h, \sigma)$ after history h and that guarantees bidder i higher payoffs than the payoffs from competitive bidding. This, however, contradicts Lemma 1.

□

Thus, for the bidders to obtain any collusive gain in a repeated standard auction, different players must have different roles. That is the continuation equilibria must depend on which player wins the object in a given period. Naming in this sense is essential.

We next show, by example, that in finitely repeated auctions attainability does restrict the players ability to collude. Consider sealed-bid second-price auctions with independent private values. The next result shows that in a large class of distributions the symmetric competitive equilibria are payoff-dominated by the asymmetric bid rotation equilibria.[8]

Proposition 2 *Consider the one-shot second-price sealed-bid auction. Suppose that*

$$\int_0^{v^h} F(v)dv \leq \left(1 - \left(\frac{N-1}{N}\right)^{N-1}\right) v^h.$$

Then the payoff in the (asymmetric) bid rotation equilibrium, π_{BR}, is strictly greater than the payoff in the (symmetric) competitive equilibrium, π_{CE}.

Note that for any $N \geq 2$ this condition is satisfied for the uniform distribution and for any distribution that first-order stochastically dominates the uniform distribution. Notice also that this condition will be satisfied for large N provided

$$\int_0^{v^h} F(v)dv < \left(1 - \frac{1}{e}\right) v^h.$$

[8]While true for a large class of distributions, the result does not hold for all distributions. Competitive bidding may be more attractive than bid rotation if the probability of high payoffs is small. Consider the case of two bidders whose valuations have a distribution $\tilde{F}$ that assumes the value $1 - \epsilon$ except on small intervals. One checks easily that if these intervals are small enough, then the expected payoff from competitive bidding exceeds the payoff from bid rotation as long as $1 - \epsilon > \frac{1}{2}$.

Proof: The payoff to a bidder in the symmetric equilibrium is

$$\pi_{CE} = \int_0^{v^h} \left[vF(v)^{N-1} - \int_0^v t(N-1)F(t)^{N-2}f(t)dt \right] f(v)dv.$$

Integrating by parts,

$$\begin{aligned}\pi_{CE} = &\int_0^{v^h} vF(v)^{N-1}f(v)dv - \left[\int_0^v t(N-1)F(t)^{N-2}f(t)dtF(v) \right]_0^{v^h} \\ &+ \int_0^{v^h} v(N-1)F(v)^{N-2}f(v)F(v)dv. \\ = &\int_0^{v^h} vNF(v)^{N-1}f(v)dv - \int_0^{v^h} v(N-1)F(v)^{N-2}f(v)dv.\end{aligned}$$

Integrating by parts again, we get

$$\pi_{CE} = \int_0^{v^h} \left[F(v)^{N-1} - F(v)^N \right] dv.$$

It is easily verified that the integrand is maximized where $F(v) = \frac{N-1}{N}$. Therefore,

$$\pi_{CE} < \left(\frac{N-1}{N} \right)^{N-1} \left(\frac{1}{N} \right) v^h.$$

On the other hand,

$$\begin{aligned}\pi_{BR} &= \frac{1}{N} \int_0^{v^h} vf(v)dv \\ &= \frac{1}{N} \left[vF(v) \right]_0^{v^h} - \frac{1}{N} \int_0^{v^h} F(v)dv \\ &= \frac{1}{N} v^h - \frac{1}{N} \int_0^{v^h} F(v)dv.\end{aligned}$$

Thus,

$$\left(\frac{N-1}{N} \right)^{N-1} v^h \leq v^h - \int_0^{v^h} F(v)dv \Rightarrow \pi_{CE} < \pi_{BR}.$$

□

Given this result, in order to show that attainability is a binding constraint in the one-shot game, it suffices to show that the competitive equilibrium is the only symmetric equilibrium in the one-shot game. This, and more, follows from the complete characterization of the set of Nash equilibria in second-price sealed-bid auctions in Blume and Heidhues [2001]. For convenience, the result for the case of at least three bidders is stated below:

Proposition 3 *Consider the second-price sealed-bid auction with independent private values and $N \geq 3$ bidders. Suppose the distributions F_i, $i = 1, \ldots, N$, of valuations have positive densities f_i on the common support $[0, v^h]$. A strategy profile is a Nash equilibrium if it satisfies: There is a $\hat{b}$ such that*

1. *any bidder with value $v > \hat{b}$ bids her value,*

2. *if $\hat{b} < v^h$ then there is a single bidder who bids at $\hat{b}$ whenever her value v satisfies $v < \hat{b}$, and if $\hat{b} \geq v^h$ then there is a single bidder who bids at or above $\hat{b}$ for any value v,*

3. *all other bidders bid 0 whenever their value v is in $[0, \hat{b})$.*

Conversely, every equilibrium satisfies these conditions up to changes of the bid functions on a set of measure zero of buyers' valuations.

One implication of this result is that the competitive equilibrium is the unique symmetric equilibrium in the one-shot game. In conjunction with Proposition 2, this implies that there is a nontrivial set of distributions for which attainability constrains the payoffs that are available in equilibrium in the one-shot game.

The second implication of this result is that attainability is a constraint in intertemporal settings, regardless of the discount factor. To see this, consider the two-period second-price auction with three or more bidders. The reason is that only winners get named. Therefore in the second period all the non-winning bidders must employ symmetric strategies. If the winning bidder is singled out in the second period, than she must be the one with a mass point at some $\hat{b} \in (0, v^h)$. One easily checks that irrespective of the the choice of $\hat{b}$ the single asymmetric bidder has higher payoff than in the symmetric equilibrium, and the symmetric bidders have lower payoffs than in the symmetric equilibrium. Thus, the only possible consequence from winning is to be rewarded in the second period. But this would raise initial bids to exactly the point where the possible gains from second-period collusion would be eliminated.

The third implication is that with a sufficiently low discount factor, attainability is a constraint on payoffs from collusion in the infinitely repeated second-price auction. With a small discount factor, first period bidding can only slightly deviate from competitive bidding.

Then Proposition 2 implies that there is a first-period loss relative to bid rotation and if the discount factor is small enough this shortfall can never be recovered in the remainder of the game.

4 Collusion with Prior Communication

In this section, we study collusion when players are not subject to attainability constraints. Intuitively, one may interpret this as allowing the players to meet and agree on a strategy-profile before the beginning of the game. Our reason for revisiting this problem, which has already been addressed by SH in the same environment, is threefold. First, we show that in an infinitely repeated standard auction there exists a sequence of equilibria with expected gains over bid rotation that grow without bound as $\delta \to 1$. Second, we show that a similar result holds for the first-price and the all-pay auction vis-a-vis competitive bidding. Third, we show a similar result vis-a-vis any collusive (or asymmetric) equilibrium of the one-shot second-price auction. These results will be used in the next section to show that for high enough discount factors one can construct attainable equilibria with higher expected payoffs than from bid rotation, competitive bidding, or repeated play of the best static equilibrium.

We begin with a preliminary result, constructing a class of bid functions for standard auctions that improve on bid rotation. Let $b^e(v, c)$ be a competitive equilibrium bid function for the environment in which each player's value is decreased by the fixed amount c and the action rules require each player to submit a bid. Note that the maximum expected payment in equilibrium is bounded by $v^h - c$. Now consider all players using the strategy $b^e(v, c)$ in the original game with unmodified payoffs. Define $v^e := \int_0^{v^h} v f(v) dv$.

Lemma 2 *If all bidders use the strategy $b^e(v, c)$, then here exists a value of c such that the expected payoff for each bidder exceeds the expected payoff from bid rotation.*

Proof: For $b^e(v, c)$, the expected payoff to a representative bidder is bounded from below by

$$g(c) := \frac{1}{N} F(c)^N \frac{\int_0^c v f(v) dv}{F(c)} + F(c)^{N-1} [F(v^h) - F(c)] \frac{\int_c^{v^h} [v - v^h + c] f(v) dv}{[F(v^h) - F(c)]}$$

$$+[1-F(c)^{N-1}](c-v^h),$$

since any bidder's expected payment is bounded from above by $v^h - c$. Observe that $g(v^h) = \frac{v^e}{N}$. We are left to show that a small decrease in c below v^h increases $g(c)$.

$$\frac{\partial g(c)}{\partial c} = \frac{N-1}{N}F(c)^{N-2}f(c)\int_0^c vf(v)dv + \frac{1}{N}F(c)^{N-1}cf(c)$$

$$+(N-1)F(c)^{N-2}\int_c^{v^h}[v-v^h+c]f(v)dv - F(c)^{N-1}[2c-v^h]f(c)$$

$$+F(c)^{N-1}\int_c^{v^h}f(v)dv - (N-1)F(c)^{N-2}(c-v^h)+[1-F(c)^{N-1}].$$

Hence,

$$\left.\frac{\partial g(c)}{\partial c}\right|_{c=v^h} = f(v^h)\left\{\frac{N-1}{N}(v^e-v^h)\right\} < 0.$$

□

Lemma 2 states that there exists a bid function of the form $b^e(v,c)$ for which the expected value is higher than the expected value of a priori randomly choosing a winner who gets the object for free.

Next, we show that there exist some $\underline{\delta}$ such that for all $\delta \geq \underline{\delta}$ there exists a perfect public equilibrium in which players gain over and above bid rotation.

Lemma 3 *For any infinitely repeated standard auction, there exist $\underline{a} > 0$, $\underline{K} > 1$, $\underline{\delta} \in (0,1)$ such that for all $\delta \geq \underline{\delta}$ there exists a perfect public equilibrium with each bidder's expected payoff at least as large as the expected bid rotation payoff in every period and larger than the expected bid rotation payoff plus $\underline{a}$ at least once every $\underline{K}$ periods.*

Proof: Suppose that competitive bidding yields a higher payoff than bid rotation. Then Lemma 3 is trivially true. Thus, assume that the payoff of bid rotation is greater than or equal to the payoff of competitive bidding. Let $b^e(v,c)$ be a bid function that - if used by all players - yields a higher payoff than randomly assigning the good for free in the one-period problem. We construct an equilibrium in which along the equilibrium path play alternates between exclusionary and non-exclusionary phases. In any non-exclusionary phase one bidder, say i, bids according to $b^e(v,c)$; the other players bid according to the function $b^e(v,c)$ for all $v \geq c$ and do not submit a bid for all $v < c$. In any exclusionary phase, the winner from the last non-exclusionary phase is excluded and the remaining bidder engage in bid rotation. Let the pair $(\alpha(c,\delta), K(c,\delta))$ be a solution to

$$c = \{\sum_{j=1}^{K-1} \delta^j + \delta^K \alpha\} \frac{v^e}{N-1},$$

for $\alpha \in [0,1)$. A player who wins the object in a non-exclusionary period is excluded for the following $K-1$ periods with certainty and is excluded in the Kth period with probability α. If no player wins the object in a non-exclusionary period, then the player i is excluded. If the excluded player wins the object in a exclusionary phase, players revert to competitive bidding ever after. Since the above strategies alternate between phases in which their payoff is higher than under bid rotation and phases in which they play bid rotation, they lead to a payoff which is higher than the expected payoff from bid rotation.

To see that these strategies form perfect public equilibria, first, observe that in the exclusionary phase no player other than the chosen bidder has an incentive to bid, since submitting a bid can only increase his current period payoffs if he wins the object. In this case, however, he forgoes all future benefits of collusion, which for high enough δ are greater than the one-period gain from deviating. Second, observe that the continuation payoff of any player who wins in a non-exclusionary phase are c below the continuation payoff of not winning the object. Thus, bidding $b^e(v,c)$ for all $v \geq c$ is a best response for any player since $b^e(v,c)$ is an equilibrium bid function in the one shot problem in which players' values are reduced by c. For any player $j \neq i$, it is optimal not to submit a bid for values below c because in this case the current period gains in case of winning is less than the reduction in his continuation payoff. For player i, however, it is optimal to bid 0 since his continuation payoff is not affected by whether he submits a bid or not. A zero bid ensures that he gets the current period gains if no other player submits a bid (if some other player submits a bid he is indifferent between not bidding and submitting a zero bid). $\square$

We are now ready to show that there always exist collusive equilibria in which the players' expected payoffs are not only greater than the payoffs of bid rotation (which was shown in SH) but that there exists a sequence of equilibria with expected gains over bid-rotation that grow without bound as $\delta \to 1$. This result is surprising since McAfee and McMillan [1992] show that in a static model without side-payments bid rotation is the optimal form of collusion for a large class of distributions. McAfee and McMillan's result does not hold in our dynamic model, because bidders can condition their future bidding behavior on the history of wins and thereby establish implicit side-payment scheme via choosing between different continuation equilibria. Intuitively, bid-rotation is extreme because it minimizes bidding competition at the cost of completely ignoring allocative efficiency. In a bid rotation scheme a player cannot win the object even if he has exceptionally high value for the good - unless it happens to be his turn to win anyhow. The bid-rotation scheme is always beaten by a collusive scheme in which players act as in a bid-rotation scheme, but are allowed to bid in case they have exceptionally high values for the object being sold. The collusive scheme is made incentive compatible by excluding a player who won when it wasn't his turn from the bid rotation for a number of periods (if an excluded player wins all bidders simply revert to competitive bidding). If the range of high values for which bidders may bid is small enough, the collusive scheme leads to an increase in allocative efficiency without leading to significant price increases.

Proposition 4 *In any infinitely repeated standard auction, there exists a sequence of perfect public equilibria such that as* $\delta \to 1$, *the difference in expected payoffs between these equilibria and bid rotation grows without bound.*

Proof: Consider the equilibria whose existence is asserted in Lemma 3. The difference D in the expected payoff of any such equilibrium and a bid rotation equilibrium satisfies:

$$D \geq \sum_{\tau=1}^{\infty} \delta^{\tau \underline{K}} \underline{a}$$

Hence, in the limit as $\delta \to 1$, $D \to \infty$. □

If bid-rotation leads to higher payoffs than competitive bidding, the above result implies that for any standard auction there exists a sequence of collusive equilibria in which the players' expected gain over competitive bidding grows without bound. We will now show that there always exists a sequence of collusive equilibria in which the players' expected gain over competitive bidding grows without bound in first-price and all-pay auctions. For second-price auctions the same result can be proven, using, very simple, attainable strategies; we postpone that discussion until the next section, which focuses on attainability.

To show that the players' expected gain over competitive bidding grows without bound in first-price and all-pay auctions, we construct equilibria in which play alternates between a non-exclusionary regime and an exclusionary regime on the equilibrium path. The winner in a non-exclusionary regime faces the risk of being excluded in the next period. This lowers the winning bidders continuation value, leads thereby to less aggressive bidding behavior, and hence to lower contemporaneous prices. This also, however, induces both a contemporaneous allocative inefficiency and a future allocative inefficiency. The contemporaneous allocative inefficiency arises because bidders have no incentive to bid positively for low values. To ensure that the contemporaneous allocative inefficiency is sufficiently low, we designate a default bidder who is also excluded if no one wins the object; this ensures that he always bids zero, gets the object for free if all bidders happen to have low values, and thereby bounds the contemporaneous inefficiency. The future allocative inefficiency can not be avoided, because the winning bidders needs to be treated differently in the future, even though his value is drawn from the same distribution. In an exclusionary period of the equilibrium we construct, all but one excluded bidder bid competitively; since competitive bidding with less bidders yields higher expected payoffs, non-excluded bidders receive an implicit side-payment from the last period's winner. We show that for high enough discount factors, the benefit of our collusive scheme through the expected reduction in the contemporaneous price always overcompensates the two types of efficiency loses in the first- and in the all-pay auction. In particular, we show that for these auctions there exists a sequence of collusive equilibria in which the players' expected gain over competitive bidding grows without bound.

Lemma 4 *In any first-price auction in which values are reduced by c, $0 < c < v_h$, there exists a symmetric equilibrium. For positive net payoffs, $v - c > 0$, the equilibrium bid function has the form*

$$b^e(v,c) = (v-c) - \frac{\int_c^v F^{N-1}(t)dt}{F^{N-1}(v)}.$$

The proof relies on standard arguments and is therefore omitted. For any first-price sealed bid auction define $\phi_\epsilon(c) := \min_{v \geq \epsilon}\{b^e(v,0) - b^e(v,c)\}$. $\phi_\epsilon(c)$ is the minimum reduction in equilibrium bids for bidders with value in the range $[\epsilon, v_h]$ if their values are reduced by c.

Lemma 5 *In any first-price sealed-bid auction $\phi_\epsilon(0)' = 1, \forall \epsilon > 0$.*

Proof: From Lemma 4, $\phi_\epsilon(c) = c - \frac{\int_0^c F^{N-1}(t)dt}{F^{N-1}(\epsilon)}$. Hence,

$$\phi_\epsilon(c)' = 1 - \left(\frac{F(c)}{F(\epsilon)}\right)^{N-1} .$$

□

We are now ready to establish the following:

Proposition 5 *In any infinitely repeated first-price or all-pay sealed-bid auction, there exists a sequence of perfect public equilibria such that as $\delta \to 1$, the difference in expected payoffs between these equilibria and repeated competitive bidding grows without bound.*

Proof: Consider the first-price sealed-bid auction. We construct an equilibrium that relies on three regimes: during a non-exclusionary regime for some $c > 0$, bidders with value $v \geq c$ bid $b^e(v, c)$ and bidders with value $v < c$ do not bid except for a default bidder who submits a zero bid for $v < c$; during an exclusionary regime all but one bidder bid according to $b^e_{N-1}(\cdot, 0)$, the symmetric equilibrium bid function for $N-1$ bidders, and the excluded bidder does not bid; and in a punishment regime everyone bids $b^e(v, 0)$.

Let $\overline{w}_N$ denote a bidder's expected one-period payoff from participating in competitive bidding with N bidders. Define

$$\alpha(c, \delta) := \frac{c}{\delta \overline{w}_{N-1}} .$$

In the following we will only consider values of c for which $\alpha(c, \delta) \in [0, 1)$.

The game starts in the non-exclusionary regime. One bidder is randomly chosen to be the default player. If the game is in the non-exclusionary regime in period t, it switches to the exclusionary regime with probability $\alpha(c, \delta)$. In the exclusionary regime the last period's winner is excluded; if no player won the object last period's default player is excluded. With probability $1 - \alpha(c, \delta)$ the game remains in the non-exclusionary regime and a new default player is chosen. From the exclusionary regime the game returns with probability one to the non-exclusionary regime unless the excluded player wins the object. In the latter case the game permanently reverts to the punishment regime.

For $\epsilon > c \geq 0$, the difference D between the sum of players' expected payoffs from the above strategy and from repeated competitive bidding satisfies:

$$\begin{aligned} D &\geq F(c)^N(-c) + [1 - F(\epsilon)^N]\phi_\epsilon(c) \\ &+ \delta(1 - \alpha(c,\delta))D + \delta\alpha(c,\delta)\{(N-1)\overline{w}_{N-1} - N\overline{w}_N\} + \delta^2\alpha(c,\delta)D \quad . \end{aligned}$$

Therefore,

$$\begin{gathered} D(1 - \delta(1 - \alpha(c,\delta)) - \delta^2\alpha(c,\delta)) \\ \geq (-c)F(c)^{N-1} + (1 - F(\epsilon)^N)\phi_\epsilon(c) + \delta\alpha(c,\delta)\{(N-1)\overline{w}_{N-1} - N\overline{w}_N\}. \end{gathered}$$

Since for $c = 0$, the right-hand side of the above expression equals zero, the sign of the right-hand side for small positive c equals the sign of its derivative with respect to c evaluated at $c = 0$. This derivative equals

$$(1 - F(\epsilon)^N)\phi'_\epsilon(0) + \frac{(N-1)\overline{w}_{N-1} - N\overline{w}_N}{\overline{w}_{N-1}} > 0$$

for sufficiently small $\epsilon > 0$.

Therefore, there exist $c > 0$ and $\epsilon > c$ such that

$$D \geq \frac{(-c)F(c)^{N-1} + (1 - F(\epsilon)^N)\phi_\epsilon(c) + \delta\alpha(c,\delta)\{(N-1)\overline{w}_{N-1} - N\overline{w}_N\}}{(1 - \delta(1 - \alpha(c,\delta)) - \delta^2\alpha(c,\delta))} > 0.$$

Thus, using the definition of $\alpha(c,\delta)$

$$D \geq \frac{(-c)F(c)^{N-1} + (1 - F(\epsilon)^N)\phi_\epsilon(c) + c\frac{(N-1)\overline{w}_{N-1} - N\overline{w}_N}{\overline{w}_{N-1}}}{(1 - \delta(1 - \alpha(c,\delta)) - \delta^2\alpha(c,\delta))} > 0.$$

Hence, fixing c and ϵ, as $\delta \to 1$, $D \to \infty$.

To show that our strategy is an equilibrium strategy, it suffices to check that no bidder has a profitable one-shot deviation after any history. After any history in which the strategy prescribes competitive bidding forever after, the incentives are identical to the incentives in the one-shot game. Thus, bidding $b^e(v, 0)$ is a best reply. After any history that places bidders in a non-exclusionary regime, it is a best response for all bidders with values below c, except the default player, not to bid because their instantaneous payoff from winning v is less than the reduction in their continuation payoff, $\alpha(c,\delta)\delta\overline{w}_{N-1} = c$. For the default player it is a best response to bid zero if $v < c$ because conditional on winning the object at price zero his continuation payoff is independent of bidding or not. The zero bid thus ensures that he gets the current period's gain. For all bidders with value $v \geq c$, it is a best response to bid $b^e(v, c)$ because $v - c$ is the net-benefit of winning the object. During an exclusionary regime, it is a best response for all non-excluded players to bid $b^e_{N-1}(v, 0)$ because in equilibrium their continuation values are not affected by their bids. For the excluded player it is a best response not to submit a bid if $v \leq \delta \frac{D}{N}$. For any c such that $D > 0$, choose $\overline{\delta}$ such that for all $\delta \geq \overline{\delta}$, $v^h \leq \delta \frac{D}{N}$. Hence, the excluded player has no incentive to deviate.

We are left to show the proposition for the case of the all-pay auction. The proof for this case is similar to the first-price auction case and is therefore relegated to Lemma 9 in the Appendix. $\square$

Sometimes in standard auctions (first-price auction, 2-bidder all-pay auction, 2-bidder Plum's λ-auction) it is known that there exists a unique equilibrium in the one-shot game.[9] In Proposition 3 we showed, however, that this is not true in the second-price sealed-bid auction. In this auction there exists a continuum of collusive equilibria in the one-shot game; furthermore there exist distributions F for which the best static equilibrium requires bidders neither to rely on bid-rotation nor to bid competitively.[10] This raises the question whether there are additional gains from intertemporal collusion compared to the best static equilibrium. Indeed, we show below that the gains from intertemporal collusion over and above the best static equilibrium grow without bound in this case.

Proposition 6 *Suppose in the best equilibrium of a static second-price sealed-bid auction with $N > 2$ bidders $\hat{b} \in (0, v^h]$. Then in any infinitely repeated second-price sealed-bid auction, there exists a sequence of perfect public equilibria such that as $\delta \to 1$, the difference in expected payoffs between these equilibria and repeatedly playing the best static equilibrium grows without bound.*

Proof: Let the best static equilibrium be indexed by $\hat{b} \in (0, v^h]$. Call it a $\hat{b}$-equilibrium.

[9]See Maskin and Riley [1996], Amann and Leininger [1996], and Plum [1992] respectively, who prove these results under weaker distributional assumptions than ours.

[10]For an example see Blume and Heidhues [2001].

If $\hat{b} = v^h$ then the above proposition follows from Proposition 4. Thus, in the following we consider only $\hat{b} \in (0, v^h)$. We construct an equilibrium in which along the equilibrium path play alternates between periods in which the $\hat{b}$-equilibrium is played and improvement periods in which the expected payoff exceeds the expected payoff from the $\hat{b}$-equilibrium. We say that a player has role $\hat{b}$ in period t in which the $\hat{b}$-equilibrium is played if he is assigned to bid $\hat{b}$ for all $v \leq \hat{b}$ in period t. Observe that the expected payoff $\hat{E}(v)$ conditional on being assigned role $\hat{b}$ is greater than the expected payoff $E(v)$ of the other role.

Next, we introduce the bid functions used in the improvement periods and show that they increase the players' expected payoff in a given period s above the expected payoff of the $\hat{b}$-equilibrium. Let $0 < c \leq \hat{b}$. Let some player $\hat{\imath}$ bid $\hat{b} - c$ for all $v \leq \hat{b}$ and bid $v - c$ for all $v > \hat{b}$; furthermore, let all players $j \neq \hat{\imath}$ refrain from submitting a bid for all $v < \hat{b}$ and bid $v - c$ for all $v > \hat{b}$. Note that with this bid functions the allocation is the same as in the $\hat{b}$-equilibrium and expected payments are lower. Hence, the bidders gain from such a change in the bid functions. Next, we show that the bid functions used in the improvement periods are enforceable for sufficiently high δ.

Let $\hat{c} := \frac{\delta}{N-1}[\hat{E}(v) - E(v)]$; $\hat{c}$ is the one-period discounted difference in the value of being allowed to enter a fair lottery with $N - 1$ players in which role $\hat{b}$ is the winning prize and role $j \neq \hat{b}$ is the losing prize and the value of being assigned the losing prize with certainty.

Consider an improvement period s and set $c = \min\{\hat{c}, \hat{b}\}$. If no player wins the object in period s, players bid competitively in all following periods $t > s$. Otherwise, the winner of period s is excluded from the chance of obtaining the role of player $\hat{b}$ in period $s + 1$ with some probability α, where $\alpha = 1$ if $\hat{c} \leq \hat{b}$ and

$$\alpha = \frac{(N-1)\hat{b}}{\delta[\hat{E}(v) - E(v)]}$$

if $\hat{c} > \hat{b}$.

First, we will observe that player $\hat{\imath}$ cannot gain from either bidding below $\hat{b} - c$ or not bidding for any value $v \in [0, \hat{b}]$. Taken his rivals' strategies as given, player $\hat{\imath}$'s probability of winning and the price he pays are unaffected by whether he bids $\hat{b} - c$ or whether he bids $b \in [0, \hat{b} - c)$. We need to show that player $\hat{\imath}$ cannot gain from not submitting a bid for any value $v \in [0, \hat{b}]$. Given the equilibrium strategy of his rivals, not submitting a bid only affects his current period payoff if all other player bidders have valuations $v \in [0, \hat{b})$. In this case, however, he looses all future collusive benefits, which are greater than the current period gain for high enough δ. Also, bidding $b > \hat{b} - c$ for values $v \in [0, \hat{b}]$ is not optimal because given his rivals' equilibrium strategies, this affects his current period payoff only if one of his rivals bids above $\hat{b} - c$. In this case, however, the price he pays $p > \hat{b} - c$ and his loss in continuation payoff from winning is c, implying that $p + c > v$. Therefore he prefers not to bid above $\hat{b} - c$ for values $v \in [0, \hat{b}]$. Furthermore, it is easy to check that bidding $v - c$ for $v > \hat{b}$ is optimal for all players. It remains to check that no player $j \neq \hat{\imath}$ has an incentive to bid for values $v < \hat{b}$. In this case, a player j wins the object only if he bids above $\hat{b} - c$, in which case the price he pays $p \geq \hat{b} - c$ and his loss in continuation value is equal to c. Thus, in this case the net value of winning the object in period s is equal to $v - p - c < 0$ and therefore the player j prefers not to bid.

Hence, for high enough δ we can construct an equilibrium in which play alternates between improvement periods in which the players' expected payoff is greater than the expected payoff from the $\hat{b}$-equilibrium and periods in which their expected payoff are equal to the expected payoff of the $\hat{b}$-equilibrium. In particular, this implies that as $\delta \to 1$, the difference in expected payoff between the improvement equilibria and repeatedly playing the $\hat{b}$-equilibrium grows without bound. $\square$

Suppose in the best static equilibrium of the second-price sealed-bid auction with $N > 2$ bidders, bidders rely neither on bid-rotation nor on competitive bidding. Then, as shown in the proof above, bidders can gain additionally from intertemporally colluding without introducing any additional allocative distortion. The reason is that in any collusive static equilibrium with $N > 2$ bidders, there exist an interval of low valuations for which nobody submits a bid. Loosely speaking, if all bids are lowered by some small amount then the expected current period price is lowered without changing the current period allocation of the good. To induce bidders to lower their bids, the winner's chance of being assigned the unique asymmetric role in the next period is reduced. Since the unique asymmetric role yields a higher expected payoff, this induces players to bid less aggressively.

We are left to show that collusion is also feasible in the absence of communication between competitors.

5 Collusion without Communication

The main result in this section is that as the discount factor converges to one, the constraints imposed by attainability do not lower the average per-period payoff that can be reached in a perfect public equilibrium. This result together with the results of the previous section imply that for high enough discount factors (i) there exist attainable perfect public equilibria with payoffs greater than the payoffs from bid rotation in any standard auction and (ii) there exist attainable perfect public equilibria with payoffs greater than the payoffs from competitive bidding in the first-price and the all-pay auction. For the case of the second-price sealed bid auction, we show that there exists an attainable perfect public equilibrium that beats competitive bidding and has a simple structure. In this equilibrium, all bidders behave symmetrically except that sometimes the last period's winner is treated differently from his competitors. This together with earlier results implies that (iii) there exist attainable perfect public equilibria with payoffs greater than the payoffs from playing the best static equilibrium in the second-price sealed-bid auction. We conclude by showing that the expected payoffs in any perfect public equilibrium of the second-price sealed-bid auction are bounded away from the full collusive gain. In summary, among patient bidders, tacit collusion is as effective as collusion with communication about strategies. However, whether tacit or not, collusion need not be efficient.

Let $\varphi(\delta)$ be the average per-bidder payoffs (across bidders and time) in a perfect public equilibrium. Let φ denote a sequence of average per-period payoffs of perfect public equilibria for which the discount factor $\delta \to 1$.

Proposition 7 *In any standard auction, for any sequence of perfect public equilibrium average payoffs φ, there exists a sequence of attainable perfect public equilibrium payoffs $\hat{\varphi}$ that satisfies* $\limsup_{\delta\to 1} \hat{\varphi} \geq \limsup_{\delta \to 1} \varphi$.

Proof: Let $\overline{\varphi} := \limsup_{\delta \longrightarrow 1} \varphi$. If $\overline{\varphi}$ is less or equal to the payoff from one-period of competitive bidding, then the result holds since repeated competitive bidding is an attainable perfect public equilibrium.

Therefore, assume from now on that $\overline{\varphi}$ is strictly greater than the payoff from one-period of competitive bidding. Hence, there exists a subsequence of φ with the same lim sup such that as $\delta \to 1$, the overall gain from the collusive perfect public equilibria associated with that subsequence over and above the payoff from repeated competitive bidding grows without bound. For the remainder of the proof, we will identify φ with that subsequence.

For any $\varphi(\delta) \in \varphi$, we will look for a corresponding attainable equilibrium per-period payoffs $\hat{\varphi}(\delta)$ and thereby construct a sequence $\hat{\varphi}$. This sequence that we will construct below satisfies $\overline{\varphi} = \limsup_{\delta \longrightarrow 1} \hat{\varphi}$, and thereby proves Proposition 7.

In particular, for any $\varphi(\delta) \in \varphi$, we will construct an attainable strategy profile with per-period payoff $\tilde{\varphi}(\delta)$. For low enough δ this strategy profile may not be an equilibrium, in which case we exchange $\varphi(\delta)$ by the per-period competitive bidding payoff. For high enough δ, however, we show that the attainable strategy profile associated with $\tilde{\varphi}(\delta)$, is indeed an equilibrium. Thus, for high enough δ, we can set $\hat{\varphi}(\delta) = \tilde{\varphi}(\delta)$. We then simply verify that $\overline{\varphi} = \limsup_{\delta \to 1} \tilde{\varphi}$.

For any perfect public equilibrium strategy profile associated with $\varphi(\delta)$, we will construct an attainable strategy profile (or candidate equilibrium) that goes through three phases along the equilibrium path: a naming phase, a reward phase, and an collusive phase. The naming phase serves to remove the initial symmetry restriction, the reward phase to provide appropriate incentives for the naming phase, and the collusive phase prescribes that players play the perfect public equilibrium strategy profile associated with $\varphi(\delta)$.

If there are N bidders, the naming phase consists of $N-1$ rounds of competitive bidding with successive exclusion. Each bidder's strategy prescribes to bid competitively in period 1, for any winner of the first $N-2$ rounds to stay out for the remaining periods of the naming phase (until period N), and for the remaining bidders to bid competitively (in the auction with the remaining number of bidders), provided there has been a winner in all the previous periods and no repeat winner. If in any one of the first $N-1$ periods there was no winner, each player bids competitively forever after. If there is a multiple winner during the first $N-1$ periods, again all players immediately revert to competitive bidding forever after. If there have been $N-1$ distinct winners during the first $N-1$ periods, a reward period starts that lasts for K periods (where K is yet to be determined).

At the beginning of the reward period, a single player is randomly selected to become eligible for a reward. Each of the $N-1$ winners is equally likely to be selected. The eligible player then enters a lottery that determines whether he will actually receive the reward. The probability that an eligible player will be rewarded is a function of the period t in which he won the object. Denote this probability by $\gamma(t)$. The reward is to become the designated winner for all of the K reward periods. If during the reward phase any bidder other than the designated bidder wins, all bidders immediately revert to competitive bidding thereafter. Let $V(n)$ denote the expected value from participating in competitive bidding with successive exclusion starting with n players. Name the $N-1$ winners by the period in which they won. Name the remaining player N. Players N and $N-1$ need not be rewarded. Since player N cannot become eligible, we can simply ignore him. For player $N-1$, set $\gamma(N-1)=0$. Consider a player in period $N-j$. His continuation payoff after winning in that period equals

$$\frac{1}{N-1}\delta^{j-1}\gamma(N-j)\frac{1-\delta^K}{1-\delta}v^e+x,$$

where x represents a part of the continuation payoff that is independent of winning or losing. The continuation payoff from losing in that period equals

$$V(j)+\frac{1}{N-1}\delta^{j-1}\frac{1}{j}\sum_{k=1}^{j-2}\gamma(N-j+k)\frac{1-\delta^K}{1-\delta}v^e+x.$$

We have to be able to assign a value $\gamma(N-j)$ with $0\leq\gamma(N-j)\leq 1$ that equates the continuation payoffs. This can be done if

$$\frac{1}{N-1}\delta^{j-1}\frac{1-\delta^K}{1-\delta}v^e\geq V(j)+\frac{1}{N-1}\delta^{j-1}\frac{j-2}{j}\frac{1-\delta^K}{1-\delta}v^e,$$

which is equivalent to

$$\frac{1}{N-1}\frac{2}{j}\delta^{j-1}\frac{1-\delta^K}{1-\delta}v^e\geq V(j).$$

This condition holds for large enough δ and K since

$$\lim_{\delta\to 1}\frac{1-\delta^K}{1-\delta}=K.$$

Finally, let us check that indeed after any history, players have an incentive to conform with the prescribed strategy for large enough discount factors. First, during the naming face, conditional on other bidders using the prescribed strategy, any currently excluded bidder prefers not to enter a bid. She can only gain from bidding if she wins. However by doing so she causes everyone to bid competitively forever after instead of colluding after period $N-1+K$. Since $N-1+K$ is finite and the gains from the collusive perfect public equilibria associated with φ increase without bound as $\delta \to 1$, there exists a $\underline{\delta}$ such that for all $\delta \geq \underline{\delta}$ she prefers not to enter a bid. Second, since for a currently included player the continuation payoffs from winning and losing have been set equal, the incentives are exactly as in the one-shot auction, and thus competitive bidding (taking into account the number of included players) is optimal. Third, during the reward phase, we only have to provide incentives for the excluded players. None of the excluded bidders has an incentive to submit a bid because, should he win, the gains are limited to a finite number of periods (less than K) whereas she loses the benefit of collusion, whose size can be made arbitrarily large. Fourth, the collusive strategy used on the equilibrium path after period $N-1+K$ is a continuation equilibrium since it is the strategy profile associated with $\varphi(\delta)$. Finally, the prescribed behavior off the equilibrium path forms an equilibrium after any history that is off the equilibrium path, since either the strategy prescribes repeated competitive bidding or we are in a history reached in which players play the continuation equilibrium $\varphi(\delta)$, and we know that the continuation equilibrium prescribes an equilibrium following that history.

Thus, we have verified that for high enough δ, the strategy profile $\tilde{\varphi}(\delta)$, is indeed an attainable perfect public equilibrium. We are left to show that $\overline{\varphi} = \limsup_{\delta\to 1} \hat{\varphi}$. For high enough δ, the difference between the payoffs of the equilibria associated with $\varphi(\delta)$ and $\tilde{\varphi}(\delta)$ is bounded by

$$(\frac{1}{1-\delta} - \frac{\delta^{N-1+K}}{1-\delta})\ \varphi(\delta),$$

and hence the difference in per-period payoffs is bounded by

$$(1-\delta^{N-1+K})\varphi(\delta),$$

which goes to zero as $\delta \to 1$. □

In the proof of Proposition 7, we construct "naming equilibria" that go through three phases: a naming phase, a reward phase, and a collusive phase. During each period of the naming phase a different bidder wins the object. Bidders are named by the period in which they win the object and winners abstain from bidding for the remainder of the naming phase. The reward phase serves to ensure participation of all the unnamed bidders at any stage of the naming phase. In the above construction we rely on a public randomization device during the reward phase in order to provide proper incentives in the naming phase.

One can construct such naming equilibria without making use of public randomization devices during the naming phase and reward phase. The construction is as follows: During the reward phase the first $n-2$ named bidders are rewarded in reverse order. The last named bidder, the winner of an auction with two bidders, need not be rewarded. The second to last winner from the naming phase, the winner from the auction with three remaining bidder, is rewarded with being the sole bidder for ν_3 periods at the beginning of the reward phase. More generally, the winner from the auction with n remaining bidders is rewarded with ν_n periods of being the sole bidder following the reward for the winner of the auction with $n-1$ periods. We will refer to ν_n as the "reward" to the winner of the auction with n remaining bidder.

For any standard auction, recall that v^e is the expected payoff to the winner from bid rotation in the static auction. Therefore, the expected present value of the reward to the winner of the auction with n remaining bidders is given by

$$r_n(\nu_3, \ldots, \nu_n; \delta) := \delta^{(n-1+\nu_3+\ldots+\nu_{n-1})} \left(\sum_{k=0}^{\nu_n - 1} \delta^k v^e \right).$$

Conditional on reaching the reward phase, for sufficiently high discount factors, any reward pattern can be enforced with the threat of reversion to competitive bidding. To construct proper incentives in the naming phase, we proceed recursively:

Let E^2 denote the expected *ex ante* value from participating in the auction with two bidders in the competitive equilibrium. Bidding behavior in the auction with three remaining bidders is based on valuations $v + r_3(\nu_3; \delta) - \delta E_2$. Note that for δ close to one and sufficiently large ν_3 we have $r_3(\nu_3; \delta) > \delta E_2$. Thus valuations are increased by a common fixed amount and by assumption the resulting auction has a unique symmetric equilibrium with a monotonic bid function, which guarantees that all three bidders participate and one gets named. Given that equilibrium, there is a well-defined *ex ante* value from participating in the three-bidder auction knowing that non-winners participate in the two-bidder auction. Denote this value by $E^3(\nu_3; \delta)$. Then bidding in the auction with four remaining bidders is based on valuations $v + r_4(\nu_3, \nu_4; \delta) - \delta E^3(\nu_3; \delta)$. $E^3(\nu_3; \delta)$ is bounded, whereas $r_4(\nu_3, \nu_4; \delta)$ can be increased without bound by choosing δ close enough to one and ν_4 sufficiently large. Therefore, there exists a value of ν_4 and $\underline{\delta}$ such that $r_4(\nu_3, \nu_4; \delta) - \delta E^3(\nu_3; \delta) > 0$ for all $\delta > \underline{\delta}$. Hence, there exists a well-defined *ex ante* value from participating in the auction with four remaining players. Denote this value by $E^4(\nu_3, \nu_4; \delta)$.

Proceeding recursively, we can find $\nu_3, \ldots, \nu_n$, and $\underline{\underline{\delta}}$ such that

$$E^n(\nu_3, \ldots, \nu_n; \delta)$$

are well defined values for participating in the auction with n remaining bidders, and

$$r_n(\nu_3, \ldots, \nu_n; \delta) > E^{n-1}(\nu_3, \ldots, \nu_n; \delta) \quad \forall \delta > \underline{\underline{\delta}}.$$

It should be noted that we can exchange perfect public equilibria with perfect Bayesian equilibria in Proposition 7 and its proof.[11] Hence, the insight that sufficiently patient bidders do not significantly suffer from language constraints is robust. We obtain the following two immediate corollaries concerning collusion without communication.

Corollary 1 *For any infinitely repeated standard auction, bid-rotation is never the optimal form of tacit collusion for sufficiently patient bidders.*

Proof: The corollary follows from Propositions 4 and 7. □

Corollary 2 *In any infinitely repeated first-price or all-pay sealed-bid auction, if bidders are sufficiently patient there exists an attainable perfect public equilibrium with an expected payoff that exceeds the expected payoff from competitive bidding.*

Proof: The corollary follows from Proposition 5 and 7. □

Another interpretation of the corollary is that there exists gains from intertemporal collusion relative to the best attainable equilibrium in the one-shot game. Because competitive

[11] Formally, however, one needs to (straightforwardly) adapt Definition 2 to allow players' strategies to also depend on their private history.

bidding is the unique equilibrium in any one-shot first-price or 2-bidder all-pay auction, the above corollary implies for these cases also that the average gains from collusion without communication about strategies are higher than from collusion with prior communication (but without side-payments) in the one-shot game.

The above results relied on a elaborate language constructions at the beginning of the game. This is not always necessary. In the second-price auction, for example, bidders only need to be able to single out the last winner in order to tacitly collude.

Proposition 8 *In any infinitely repeated second-price sealed-bid auction in which bidders are sufficiently patient, there exists a name-simple equilibrium that payoff dominates repeated competitive bidding.*

Proof: We construct an equilibrium that relies on three regimes: during a non-exclusionary regime for some $c > 0$, bidders with value $v \geq c$ bid $v - c$ and bidders with value $v < c$ do not bid, during an exclusionary regime all but one player bid their value and the excluded player does not bid, and in a punishment regime everyone bids their value.

Let $\overline{w}_N$ denote a player's expected one-period payoff from participating in competitive bidding with N players. Let $W(c)$ denote the expected value from the equilibrium we are about to construct. Define

$$\alpha(c, \delta) := \frac{c}{\delta\{[1 - F(c)^{N-1}]\overline{w}_{N-1} + F(c)^{N-1}(1-\delta)W(c)\}} \quad .$$

Note that $\alpha(c, \delta)$ is well-defined for small c because as c converges to zero, the denominator converges to $\delta\overline{w}_{N-1}$. In the following, we will only consider values of c for which $\alpha(c, \delta) \in (0, 1)$.

The game starts in the non-exclusionary regime. If the game is in the non-exclusionary regime in period t, it switches to the exclusionary regime with probability $\alpha(c, \delta)$, if someone wins the object. In the exclusionary regime the last winner is excluded. With probability $1 - \alpha(c, \delta)$ the game remains in the non-exclusionary regime. From the exclusionary regime the game returns with probability one to the non-exclusionary regime unless the excluded player wins the object. In the latter case the game permanently reverts to the punishment regime. If no player wins the object in a non-exclusionary regime, the game remains in the non-exclusionary regime.

The difference D between all players' expected payoffs from the above strategy and repeated competitive bidding satisfies:

$$\begin{aligned} D &\geq F(c)^N(-c) + [1-F(c)^{N-1}]c + \delta F(c)^N D \\ &+ \delta[1-F(c)^N]\Big\{(1-\alpha(c,\delta))D + \alpha(c,\delta)\{(N-1)\overline{w}_{N-1} - N\overline{w}_N\} + \delta\alpha(c,\delta)D\Big\} \quad . \end{aligned}$$

Therefore,

$$\begin{aligned} & D\Big\{1-\delta F(c)^N - \delta[1-F(c)^N]\{1-\alpha(c,\delta)(1-\delta)\}\Big\} \\ & \geq (-c)(F(c)^N + F(c)^{N-1}) + c + \alpha(c,\delta)\delta[1-F(c)^N]\{(N-1)\overline{w}_{N-1} - N\overline{w}_N\} \\ & \geq \alpha(c,\delta)\delta\Big\{\{[1-F(c)^{N-1}]\overline{w}_{N-1} + F(c)^{N-1}(1-\delta)W(c)\}(-2F(c)^{N-1}) \\ & +[1-F(c)^{N-1}]\overline{w}_{N-1} + F(c)^{N-1}(1-\delta)W(c) \\ & +[1-F(c)^N](N-1)\overline{w}_{N-1} - [1-F(c)^N]N\overline{w}_N\Big\} \quad . \end{aligned}$$

One easily checks that for small c the expression on the right hand side of the above inequality is positive. Therefore, $D > 0$.

To show that our strategy is an equilibrium strategy, it suffices to check that no bidder has a profitable one-shot deviation after any history. After any history in which the strategy prescribes competitive bidding forever after, the incentives are identical to the incentives in the one-shot game. Thus bidding one's value is a best reply. After any history that places bidders in a non-exclusionary regime, it is a best response for all bidders with values below c not to bid because their instantaneous payoff from winning v is less than the reduction in their continuation payoff, $\alpha(c,\delta)\delta\{[1-F(c)^{N-1}]\overline{w}_{N-1} + F(c)^{N-1}(1-\delta)W(c)\} = c$. For all bidders with value $v \geq c$, it is a best response to bid the net gain of winning the object, $v-c$. During an exclusionary regime, it is a best response for all non-excluded players to bid their values because in equilibrium their continuation values are not affected by their bids. For the excluded player it is a best response to not submit a bid if $v \leq \delta\frac{D}{N}$. For any c such that $D > 0$, there exists a $\overline{\delta}$ such that for all $\delta \geq \overline{\delta}$, $v \leq \delta \frac{D}{N}$ as can easily be checked from the inequality above.

The strategy profile is name-simple because all bidders behave symmetrically except for an excluded bidder in an exclusionary regime; this bidder however was the last period's winner and is thereby differentiated (named). □

Thus, we showed that simple collusive schemes can be effective. The above result also enables us to state the following:

Corollary 3 *In any infinitely repeated second-price sealed-bid auction with $N > 2$ sufficiently patient bidders, there exists an attainable perfect public equilibrium with an expected payoff that exceeds the expected payoff from repeatedly playing the best static equilibrium.*

Proof: If players bid competitively in the best static equilibrium, the corollary follows from Proposition 8. Otherwise, the corollary follows from Proposition 6 and 7. □

Thus, in the second-price auction the average gains from intertemporal collusion without communication are higher than from collusion with communication (but without side-payments) in a one-shot interaction.

Knowing that attainability does not impose significant constraints on collusion among sufficiently patient bidders, one may ask whether there are attainable perfect public equilibria that sustain efficient collusion in our environment. Proposition 7 implies that efficient collusion in attainable perfect public equilibria can be approximated if and only if there is such an approximation without the attainability requirement. This turns out to be impossible for second-price auctions. We will demonstrate the impossibility of efficient collusion in attainable perfect public equilibria by establishing an anti-folk-theorem: Even without the attainability constraint, collusive payoffs are bounded away from the efficient frontier. This is a consequence of the informational limitations in our environment.

We say that bidders achieve the *full collusive gain* in a given period, if the bidder with the highest valuation receives the object at a zero price. In contrast, the bidders' expected payoff is bounded away from the full collusive gain if assigning the object with high probability to the highest-valuation bidder requires that the expected price is significantly above zero. Formally, let $i(t) \in \arg\max_i\{v_i(t)\}$, and σ a strategy profile in the repeated auction. For any probability K, we say that the allocation induced by the bid functions $b_i(v_i(t), h(t), \sigma),\ i = 1, \ldots, N$ is *K-efficient* in the period following history $h(t)$ if

$$\text{Prob}\{i(t) = \arg\max_i b_i(v_i(t), h(t), \sigma)\} \geq K.$$

Given the bid functions $b_i(v_i(t), h(t), \sigma),\ i = 1, \ldots, N$, there will be an expected value of the second order statistic of bids. Refer to this expected value of the second highest bid as the *expected price.*

Definition 6 *Bidders' expected payoffs are bounded away from the full collusive gain if there exists $\underline{K} < 1$ and $\underline{p} > 0$ such that in any period t following history $h(t)$ in which the allocation that is induced by $b_i(v_i(t), h(t), \sigma),\ i = 1, \ldots, N$, is K-efficient with $K \geq \underline{K}$, the expected price p satisfies $p > \underline{p}$.*

Proposition 9 *In any perfect public equilibrium of the second-price sealed-bid auction with $N \geq 3$ bidders, bidders' expected payoffs are bounded away from the full collusive gain.*

The proof is somewhat technical and therefore relegated to the appendix. The intuition for the result however is simple: Near efficiency requires that bid functions are similar. As a consequence expected continuation values from winning vary little with the level of one's own bid. A low expected price requires low bids, including low bids at high valuations. Bidders will only bid low at high valuations if winning causes a sufficient loss in expected continuation value. Hence, if the expected continuation value from winning doesn't vary much with the bid, bidders with low and moderate valuations strictly prefer to refrain from bidding. This contradicts near efficiency.

Observe that the Proposition (and its proof) extends to the case of two bidders as long as bidders condition their behavior on $h(t)$ only. In this special case, however, the second highest bid in all past periods is common knowledge and in a PPE players could condition their behavior also on these past bids.

6 Related Literature

In this section, we discuss three strands of related literature. First, we relate our work to papers on strategic uncertainty, especially attainability. Second, we compare our paper to other papers on tacit collusion in auctions and industrial organization. Finally, we discuss papers on infinitely repeated games with imperfect observability.

Strategic uncertainty has been a concern in game theory for a long time. For this reason standard textbooks (see for example Moulin [1986, p.106] or Kreps [1990, p.411]) frequently describe Nash equilibria as self-enforcing agreements. One envisions that players meet before the game to engage in pre-play communication. For any agreement emerging from such a meeting to be stable, it will have to be a Nash equilibrium. However, as Luce and Raiffa [1957, p.172] point out, this perspective on how players might coordinate their expectations gets around the fact that "... the equilibrium notion does not serve in general as a guide to action." Nevertheless, it is a matter of practical concern, how agents in a strategic situation will choose their course of actions without the benefit of pre-play communication, or some other coordinating mechanism. While the symmetry restrictions imposed by attainability may not provide a definitive answer to this puzzle, they operationalize constraints that players face when they cannot use communication to coordinate their strategies.[12]

Crawford and Haller [1990], henceforth CH, define attainable strategies and use them to study two-player games with complete symmetry (i.e. a complete absence of a common language) and common interest.[13] Blume [2000] extends CH's analysis to allow for partial structure, interpreted as prior understandings (similar to grammar in language), that aid players ability to coordinate. In Section 3, we show how such prior understandings can be incorporated into our model of tacit collusion. Bhaskar [2000] applies CH's approach to a game with conflict, the infinitely repeated *Battle of the Sexes.* Our paper is related to Bhaskar in that we use idea of attainability in repeated game with a conflict of interest. In the repeated n-bidder auction setting we study, however, the problem is compounded by the players' private information.

That the history of the game enables players to overcome initial symmetry has been shown in a number of related experimental papers. Blume, DeJong, Kim and Sprinkle [1998] and [2001] experimentally study the emergence of meaning for *a priori* meaningless messages

[12] Symmetry restrictions also play a prominent role in the "rational theory of equilibrium selection" of Harsanyi and Selten [1988]. In particular, they also require their solution to be invariant to the renaming of players, actions, and choices.

[13] Kramarz [1996] extends CH's work to n-player pure coordination games. Alpern and Reyniers [2000] use attainable strategies, subject to additional Markov restrictions, to study games with many players whose goal is to disperse themselves among a finite set of locations.

in repeated sender-receiver games. In these studies the initial coordination probability in a common interest game is close to 50% and rises over time as messages acquire meaning as a result of repeated play. Similar results hold when there is some conflict of interest between senders and receivers. Blume and Gneezy [2000] find evidence for attainability in cognitively simple games. Blume and Gneezy [2001] show that the attainability idea is also useful in settings where players do not have common knowledge of the language they are using. They show that players form beliefs about each others languages and use *cognitive forward induction,* i.e. signal their language if given the opportunity.

There is also some evidence that bidders in auctions try to use the history of the game to coordinate when they are prevented from communicating directly. One example is the use of *code bidding.* In the FCC auctions bidders used the trailing digits of a bid to communicate when it was still feasible (see Cramton and Schwartz [2000]). Another example is the signaling in early rounds of an auction how to eventually split the market, as occurred in the German 1999 spectrum auction of ten licenses.[14] Here, the firms used the observable history of the game (in this case past bids) to desymmetrize the objects and coordinate on a noncooperative equilibrium in which they shared the market.

That players use substitutes for explicit, face to face communication can be observed in non-auction settings, such as the airline market, as well. The major US airline companies compete in hundreds of city pairs, with changing costs and demand conditions, over time. To successfully coordinate on a collusive equilibrium in the absence of any explicit communication therefore seems a formidable task. From 1988-1992 airlines used the ATP fare system to exchange not only current fares but also future intended fares and, through the use of footnotes in the database, to assign different city-pair fares a common symbol. This expanded (and organized) the observable history of the market game considerably and as the DOJ argued in its Competitive Impact Statement [1994] enabled the firms to overcome uncertainty in coordinating their pricing behavior through an "electronic dialogue." In the settlement the DOJ contended that "there may be an element of communication inherent

[14] Grimm, Riedel and Wolfstetter [2001], who consulted one of the bidders, state that: "Mannesmann started with a jump bid on all frequencies for two reasons: to bring the price uniformly to the critical level at which smaller providers would quit the auction, and to coordinate efficiently with T-Mobil on how to divide the frequencies numbered from 1 to 10." They also provide a game-theoretic analysis of this auction, which, however, abstracts from the coordination issue.

in fares that are actually available and intended to be sold," which is considered legal, but that actions should be avoided that convey "other information concerning the defendant's planned or contemplated fares or changes to fares."[15] By restricting the information firms were allowed to publish, the settlement reduces the observable history that firms can use as a substitute for explicit, face to face communication. While this case is considerably more complex than our simple auction environment, their are obvious parallels for future work to explore.

Our paper is related to Bernheim [1984] and Pearce [1984] work on rationalizability in that rationalizability captures aspects of strategic uncertainty as well. But rather than constraining beliefs as attainability does, it forces us to be permissive. For instance, rather than ruling out a bid rotation scheme in a one-shot second-price auction, that has a particular designated bidder win the the object, it rationalizes beliefs consistent with a variety of such schemes that result from exchanging the role of the winning bidder.

Section 5, where we construct a sequence of equilibria with payoffs unboundedly higher than the payoff from bid rotation or competitive bidding, draws on and strengthens results in Skrzypacz and Hopenhayn [1999], henceforth SH. Using arguments from Abreu, Pearce and Stachetti [1990], SH provide conditions for the existence of noncooperative equilibria that payoff dominate both competitive bidding and bid rotation in a class of repeated auctions. They explicitly construct such equilibria for the case of first- and second-price auctions. We show that these gains can be made unbounded and that there need not be designated bidders at the beginning of the auctions.[16] We also differ from SH in that we investigate the restrictions imposed by the lack of explicit communication about strategies.

Aoyagi [2000] analyzes a similar environment as SH and we do in which he allows for affiliated values. Whereas we analyze the case of less communication than SH, he analyzes

[15] See the "Antitrust Division's Statement of Decree Applicability," attached to the Competitive Impact Statement concerning the United States of America vs Airline Tariff Publishing Company [1994].

[16] Recently, we became aware of a independent work by Skrzypacz and Hopenhayn [2001] that also extends their earlier results. Focusing on a two player example in which each player draws his value from an identical and symmetric distribution, they show that the average per-period gains can be substantial. While their construction is somewhat similar to ours in spirit, their results do not subsume ours for the following reasons: we do not rely on symmetric distributions, we prove our results for any number of bidders, we show that bidders can do better than bid rotation in any standard auction, we show that bidders can always gain relative to the best static equilibrium in the second-price auction, and we explicitly show that in the all-pay auction players can gain relative to competitive bidding.

a richer communication environment than SH. In his setting bidders communicate by way of a mediator to whom they report their private information and from whom they receive bidding instructions. He also shows that bidders can improve on competitive bidding and bid-rotation.

Athey, Bagwell and Sanchirico [2000] study symmetric equilibria in a repeated Bertrand game with private information. In these equilibria "firms move through collusive and war phases together." This symmetry condition is much stronger than attainability and close to the name-freeness condition that we discuss below.[17] Name-freeness rules out collusion in repeated standard auctions whereas Athey, Bagwell and Sanchirico find symmetric collusive equilibria in the repeated Bertrand game.[18]

In a related paper, Athey and Bagwell [2001] study the impact of cost announcements in a repeated Bertrand game. They model lack of communication as an inability to announce one's type in a given period and prove a Folk Theorem, which also holds for discount factors strictly less than one. They do not address the strategic uncertainty that the lack of communication creates.

Our paper is also related to the literature on infinitely repeated games with imperfect public information. We use perfect public equilibria, which were introduced by Fudenberg, Levine and Maskin [1994], henceforth FLM. One appeal of public perfect equilibria is that they are recursive and thus dynamic programming techniques apply. Under general conditions, FLM show that one can establish a Folk Theorem in perfect public equilibria for finite-action-set stage games if there is a sufficient number of public outcomes which are observable. If, however, the action set is large relative to the observable public outcomes, then a Folk Theorem may not hold. Counterexamples, that is games in which the payoffs are bounded away from the efficient payoffs for any discount factor, have been given in Radner, Myerson and Maskin [1986] and in FLM. In our setup in which only the winner is observed, we show that, no matter how patient the bidders are, their average payoffs are bounded away from collusive efficiency in the repeated second-price sealed-bid auction. We thus provide a

[17] Abreu, Pearce and Stacchetti [1986] also focus on symmetric equilibria of this kind.

[18] Interestingly, Athey, Bagwell and Sanchirico motivate their focus on symmetric strategies by arguing that they are appealingly simple. While explicitly refraining from proposing how firms coordinate on an equilibrium, they argue that the asymmetric equilibria are most plausible "when a small number of firms (...) communicate explicitly."

counterexample in an auction environment.

Intuitively, it may seem reasonable that bidders could use Radner-type review strategies to enforce collusive behavior (see Radner [1985]). We show, however, that if players condition their behavior on public information only (i.e. how often and when a player won the object), then their average payoffs are bounded away from collusive efficiency. If they are allowed to condition their behavior on private information, then it remains an open question whether all PBE-payoffs are bounded away from collusive efficiency. The problem is that repeated games with private monitoring are not well-understood.

When relying on private rather than public histories, even if one player is almost sure that another has deviated and would want to punish if he believed that others were punishing, he cannot be sure that others are almost sure that someone has deviated because there is no common knowledge of the relevant history. In particular, this implies that players play correlated rather than continuation equilibria and it is not clear whether dynamic programming techniques can be used. If players can communicate, this problem can be overcome since the players may use cheap talk to generate common beliefs about the histories, as in Compte [1998] and in Kandori and Matushima [1998]. In this paper, however, we are interested in collusion without communication and hence we cannot use the techniques applied in Compte [1998] and Kandori and Matushima [1998] to check whether all perfect Bayesian equilibria are bounded away from the full collusive gain.

7 Conclusion

In this paper, we interpreted *tacit collusion* as *collusion without communication* and adopted a stringent interpretation of "without communication." We argued that the lack of communication between players leads to strategic uncertainty, which we modelled through symmetry restrictions on their repeated game strategies. This enabled us to investigate how symmetry restrictions - introduced through the lack of communication about strategy - interact with restrictions on information about past actions. We showed that severe symmetry restrictions (requiring name-free strategies) completely rule out collusion in our informationally restrictive environment in which bids are unobservable, whereas they do not rule out collusion if past bids are observable.[19] We argued in favor of modelling strategic uncertainty through requiring players to use attainable strategies, which impose less extreme symmetry restrictions that acknowledge the possible removal of symmetry constraints through repeated interaction. We showed that attainability limits collusive payoffs with short time horizons or impatient bidders. Patient bidders, however, can overcome the symmetry constraints imposed by attainable strategies. Focusing on the repeated second-price sealed-bid auction, we showed that for patient bidders the informational constraints are more important than the constraints imposed by the strategic uncertainty, whereas the reverse often holds for impatient bidders.

Throughout, we used symmetry constraints in a symmetric game and it is only fair to ask what role, if any, we ascribe to symmetry in general. One answer is that even if there are asymmetries, the agents may have no clear sense of how to use them. In this paper, for example, we modelled bidders as initially nameless and therefore unable to use strategies that make use of players' identities. However, we interpret literal namelessness only a modelling device. In practice, even if names differ, it suffices that they do not induce natural role differences. We also assumed that players draw their values from identical distributions. Again differences in distributions may not automatically confer natural role differences, although here the problem is a bit more delicate because it is likely that any equilibrium will prescribe different behavior to bidders with different distributions.[20]

Our results indicate that, relative to standard noncooperative models, acknowledging the strategic uncertainty created by the lack of communication may severely limit the ability to collude in one-shot games or environments in which the discount factor is low. The strategic uncertainty is less important in determining players ability to collude if interactions are frequent (or discount factors high). Hopefully, our results help to build a general theory of collusion in the absence of explicit communication. Such a theory is needed to guide policies aimed at reducing communication between competitors such as the one implemented by settlement between the DOJ and major airline companies discussed in the previous section.

[19]That there is scope for name-free collusion if bidders observe past bids is obvious. Think for example of a simple trigger strategy in which players bid zero if all players have been bidding zero in the past and bid competitively otherwise. For high enough discount factors, this is an equilibrium if bid rotation yields a higher payoff than competitive bidding.

[20]This is not always the case. Take for example the second-price sealed-bid auction.

Appendix: Proofs

The following Lemmas are used to prove Lemma 9 below.

Lemma 6 *In any all-pay sealed bid auction in which values are reduced by* c, $0 < c < v_h$, *there exists a symmetric equilibrium. For positive net payoffs,* $v - c > 0$, *the equilibrium bid function has the form*

$$b^e(v,c) = (v-c)F^{N-1}(v) - \int_c^v F^{N-1}(t)dt.$$

The proof relies on standard arguments and is therefore omitted. For any all-pay sealed bid auction define $\phi_v(c) := b^e(v,0) - b^e(v,c)$. $\phi_v(c)$ is the reduction in equilibrium bids for bidders whose values v are reduced by c.

Lemma 7 *In any all-pay sealed-bid auction* $\phi_v(0)' = F^{N-1}(v)$.

Proof: From Lemma 6, $\phi_v(c) = cF^{N-1}(v) - \int_0^c F^{N-1}(t)dt$. Hence,

$$\phi_v(c)' = F^{N-1}(v) - F^{N-1}(c).$$

□

Lemma 8 *In any all-pay sealed bid auction one has,*

$$\int_0^{v_h} \phi_v(0)' f(v)dv = \frac{1}{N}.$$

Proof: Integration by parts gives that

$$\int_0^{v_h} F^{N-1}(v)f(v)dv = [F^{N-1}(v)F(v)]_0^{v_h} - \int_0^{v_h} (N-1)F^{N-2}(v)f(v)F(v)dv.$$

Hence,

$$N\int_0^{v_h} F^{N-1}(v)f(v)dv = [F^{N-1}(v)F(v)]_0^{v_h}.$$

Thus,

$$\int_0^{v_h} F^{N-1}(v)f(v)dv = \frac{1}{N}.$$

Since, by Lemma 7,

$$\int_0^{v_h} \phi_v(0)'f(v)dv = \int_0^{v_h} F^{N-1}(v)f(v)dv,$$

one has $\int_0^{v_h} \phi_v(0)'f(v)dv = \frac{1}{N}$. $\square$

Lemma 9 *In any infinitely repeated all-pay sealed-bid auction, there exists a sequence of perfect public equilibria such that as $\delta \to 1$, the difference in expected payoffs between these equilibria and repeated competitive bidding grows without bound.*

Proof: We construct an equilibrium that relies on three regimes: during a non-exclusionary regime for some $c > 0$, bidders with value $v \geq c$ bid $b^e(v, c)$ and bidders with value $v < c$ do not bid except for a default bidder who submits a zero bid for $v < c$; during an exclusionary regime all but one bidder bid according to $b^e_{N-1}(\cdot, 0)$, the symmetric equilibrium bid function for $N-1$ bidders, and the excluded bidder does not bid; and in a punishment regime everyone bids $b^e(v, 0)$.

Let $\overline{w}_N$ denote a bidder's expected one-period payoff from participating in competitive bidding with N bidders. Define

$$\alpha(c, \delta) := \frac{c}{\delta \overline{w}_{N-1}} \quad .$$

In the following we will only consider values of c for which $\alpha(c, \delta) \in [0, 1)$.

The game starts in the non-exclusionary regime. One bidder is randomly chosen to be the default player. If the game is in the non-exclusionary regime in period t, it switches to the exclusionary regime with probability $\alpha(c, \delta)$. In the exclusionary regime the last period's winner is excluded; if no player won the object last period's default player is excluded. With probability $1 - \alpha(c, \delta)$ the game remains in the non-exclusionary regime and a new default player is chosen. From the exclusionary regime the game returns with probability one to the non-exclusionary regime unless the excluded player wins the object. In the latter case the game permanently reverts to the punishment regime.

For $\epsilon > c \geq 0$, the difference D between the sum of players' expected payoffs from the above strategy and from repeated competitive bidding satisfies:

$$\begin{aligned} D &\geq F(c)^N(-c) + N[1-F(\epsilon)]^N \int_\epsilon^{v_h} \phi_v(c) f(v) dv \\ &+ \delta(1-\alpha(c,\delta))D + \delta\alpha(c,\delta)\{(N-1)\overline{w}_{N-1} - N\overline{w}_N\} + \delta^2\alpha(c,\delta)D \ . \end{aligned}$$

Therefore,

$$D(1-\delta(1-\alpha(c,\delta)) - \delta^2\alpha(c,\delta))$$
$$\geq (-c)F(c)^N + N[1-F(\epsilon)]^N \int_\epsilon^{v_h} \phi_v(c) f(v) dv + \delta\alpha(c,\delta)\{(N-1)\overline{w}_{N-1} - N\overline{w}_N\}.$$

Since for $c = 0$, the right-hand side of the above expression equals zero, the sign of the right-hand side for small positive c equals the sign of its derivative with respect to c evaluated at $c = 0$. This derivative equals

$$N[1-F(\epsilon)]^N \int_\epsilon^{v_h} \phi'_v(0) f(v) dv + \frac{(N-1)\overline{w}_{N-1} - N\overline{w}_N}{\overline{w}_{N-1}},$$

which at $\epsilon = 0$ simplifies by Lemma 8, to

$$1 + \frac{(N-1)\overline{w}_{N-1} - N\overline{w}_N}{\overline{w}_{N-1}} > 0,$$

and hence for sufficiently small $\epsilon > 0$ the above derivative of the right hand side is strictly greater than zero.

Therefore, there exist $c > 0$ and $\epsilon > c$ such that

$$D \geq \frac{(-c)F(c)^N + N[1-F(\epsilon)]^N \int_\epsilon^{v_h} \phi_v(c) f(v) dv + \delta\alpha(c,\delta)\{(N-1)\overline{w}_{N-1} - N\overline{w}_N\}}{(1-\delta(1-\alpha(c,\delta)) - \delta^2\alpha(c,\delta))} > 0.$$

Thus, using the definition of $\alpha(c,\delta)$

$$D \geq \frac{(-c)F(c)^N + N[1-F(\epsilon)]^N \int_\epsilon^{v_h} \phi_v(c) f(v) dv + c\frac{(N-1)\overline{w}_{N-1} - N\overline{w}_N}{\overline{w}_{N-1}}}{(1-\delta(1-\alpha(c,\delta)) - \delta^2\alpha(c,\delta))} > 0.$$

Hence, fixing c and ϵ, as $\delta \to 1$, $D \to \infty$.

To show that our strategy is an equilibrium strategy for sufficiently high δ, it suffices to check that no bidder has a profitable one-shot deviation after any history. After any history in which the strategy prescribes competitive bidding forever after, the incentives are identical to the incentives in the one-shot game. Thus bidding $b^e(v, 0)$ is a best reply. After any history that places bidders in a non-exclusionary regime, it is a best response for all bidders with values below c, except the default player, not to bid because their instantaneous payoff from winning v is less than the reduction in their continuation payoff, $\alpha(c,\delta)\delta\overline{w}_{N-1} = c$. For the default player it is a best response to bid zero if $v < c$, for conditional on bidding zero, his continuation payoff is independent of bidding or not. The zero bid thus ensures that he gets the current period's gain. For all bidders with value $v \geq c$, it is a best response to bid $b^e(v, c)$ because $v - c$ is the net-benefit of winning the object. During an exclusionary regime, it is a best response for all non-excluded players to bid $b^e_{N-1}(v, 0)$ because in equilibrium their continuation values are not affected by their bids. Clearly, for the excluded player it is a best response not to submit a bid if $v \leq \delta\frac{D}{N}$. For any c such that $D > 0$, choose $\overline{\delta}$ such that for all $\delta \geq \overline{\delta}$, $v_h \leq \delta\frac{D}{N}$. Hence, the excluded player has no incentive to deviate for sufficiently high δ. □

In the remainder of the appendix, we establish some preliminary results and then prove Proposition 9. To establish Proposition 9, we will make use of the fact that for large K, K-efficiency implies a monotonicity condition on the set of strategies used by the N bidders: Let $(\omega^1, \ldots, \omega^k) := \{(0, \epsilon_1), (\epsilon_1, 2\epsilon_1), \ldots, (v^h - \epsilon_1, v^h)\}$, where $\epsilon_1 = v^h/k$ for some positive integer k. Let μ_F denote the (probability) measure induced by the distribution function F and recall that F has a positive density.

Definition 7 *An allocation induced by bid functions $b_i(v_i(t), h(t), \sigma)$, $i = 1, \ldots, N$, is said to be* (ϵ_1, ϵ_2)-monotonic *in period t following history $h(t)$, if there exists a collection of sets $\varpi = (\varpi^1, \ldots, \varpi^k)$ such that $\varpi^l \subseteq \omega^l$, the Lebesgue measure of each ϖ^l is at least $\epsilon_1(1 - \epsilon_2)$, and such that for all $v^l \in \varpi^l$ and all $v^{l+1} \in \varpi^{l+1}$, one has $b_i(v^l, h(t), \sigma) < b_j(v^{l+1}, h(t), \sigma)$, $\forall i, j$.*

We will proceed by showing that K-efficiency for large $K < 1$ implies (ϵ_1, ϵ_2)-monotonicity for small $(\epsilon_1, \epsilon_2) \gg (0, 0)$, which will be used later to show that in equilibrium continuation values do not vary substantially with one's bid.

Lemma 10 *For any $\epsilon_1, \epsilon_2 > 0$, there exists $\underline{K} < 1$ such that any K-efficient allocation induced by bid functions $b_i(v_i(t), h(t), \sigma)$, $i = 1, \ldots, N$, for $\underline{K} < K < 1$ is (ϵ_1, ϵ_2)-monotonic.*

Proof: Consider the allocation induced by bid functions $b_i(v_i(t), h(t), \sigma)$, $i = 1, \ldots, N$. For any $\epsilon_1, \epsilon_2 > 0$ and corresponding partition $\{\omega^l\}$, define

$$\alpha^{il}(\zeta) := \inf \left\{ b | \text{Prob}\{b_i(v_i(t), h(t), \sigma) \leq b | v_i(t) \in \omega^l\} \geq \zeta \right\}$$

and

$$\beta^{il}(\zeta) := \sup \left\{ b | \text{Prob}\{b_i(v_i(t), h(t), \sigma) \geq b | v_i(t) \in \omega^l\} \geq \zeta \right\}$$

as functions of $\zeta \in (0, 1)$. For sufficiently large $\underline{K}$, we must have

$$\alpha^{il}(\zeta) > \beta^{j,l-1}(\zeta) \; \forall i \neq j, \; \forall K \in (\underline{K}, 1].$$

Let $j(i) \in \arg\max_{j \neq i} \beta^{j,l-1}(\zeta)$. Then $\alpha^{il}(\zeta) > \beta^{j(i),l-1}(\zeta)$, and for every bidder, with the possible exception of bidder i, there is a set of valuations in ω^{l-1} where he bids at or below $\beta^{j(i),l-1}(\zeta)$ and that has at least probability $(1-\zeta)\mu_F(\omega^{l-1})$.

Consider bidder i and for some other bidder i' a set of valuations in ω^{l-1} where i' bids at or below $\beta^{j(i),l-1}(\zeta)$ and that has at least probability $(1-\zeta)\mu_F(\omega^{l-1})$. For most valuations in that range, i must bid at or below $\beta^{j(i),l-1}(\zeta)$ as well in order not to violate K-efficiency if $\underline{K}$ is sufficiently large. Therefore, for sufficiently large $\underline{K}$, for every bidder there is a set of valuations in ω^{l-1} that has at least probability $(1-\zeta)^2\mu_F(\omega^{l-1})$ and on which he bids below $\beta^{j(i),l-1}(\zeta)$. Let ξ solve $(1-\xi) = (1-\zeta)^2$.

Let $\hat{j} := j(i)$, and $\hat{i} \in \arg\min_{i \neq \hat{j}} \alpha^{il}(\zeta)$. Repeating the foregoing argument for α instead of β, we conclude that for sufficiently large $\underline{K}$ each bidder with value in ω^l bids at or above $\alpha^{\hat{i}l}(\zeta)$ on a set with probability at least $(1-\xi)\mu_F(\omega^l))$ and each bidder with value in ω^{l-1} bids at or below $\beta^{\hat{j},l-1}(\zeta)$ on a set with probability at least $(1-\xi)\mu_F(\omega^{l-1}))$.

Let $\alpha^l(\zeta) := \alpha^{\hat{i}l}(\zeta)$ and $\beta^l(\zeta) := \beta^{\hat{j}l}(\zeta)$. Then we have shown that for sufficiently large $\underline{K}$, for each bidder there exists a subset of ω^l with probability at least $(1-2\xi)\mu_F(\omega^l))$ on which that bidder bids in $[\alpha^l(\zeta), \beta^l(\zeta)]$. Repeatedly using the fact that for any measure μ, $\mu(E \cap F) = \mu(E) + \mu(F) - \mu(E \cup F)$, we infer that in each set ω^l there is a subset of valuations $\varpi^l \subset \omega^l$ for which all bidders bid in $[\beta^l(\zeta), \alpha^l(\zeta)]$ and that has probability greater than or equal to $(1-2n\xi)\mu_F(\omega^l))$. This will be true for any $\zeta \in (0, 1)$, for sufficiently large $\underline{K}$. Since F has a positive density everywhere, Lebesgue measure and the probability measure corresponding to F are mutually absolutely continuous. Therefore, for any $\epsilon_2 > 0$ we can find $\underline{K}$ large enough such that ζ (and thus ξ) is small enough to guarantee that the set $\varpi^l \subset \omega^l$ has Lebesgue measure at least $\epsilon_1(1-\epsilon_2)$. □

Let $b_j(\cdot, h, \sigma)$ denote bidder j's bid function that is induced by σ_j in the period following history h, and use $G_j(\cdot, h, \sigma)$ to denote the distribution of bids that is induced by $b_j(\cdot, h, \sigma)$. Lemma 10 immediately implies the following corollary:

Corollary 4 *If $\{b_j(\cdot, h, \sigma^K)\}_{j\in N}$ is a sequence of collections of bid functions, indexed by K, that induce K-efficient allocations following history h with $K \to 1$, then for each $j \in N$, the sequence of distribution functions $G_j(b_j(\cdot, h, \sigma^K), h, \sigma^K)$ converges in measure (Lebesgue and μ_F) to $F(\cdot)$, i.e. for any $\epsilon > 0$*

$$\mu\left(\{v : |G_j(b_j(v, h, \sigma^K), h, \sigma^K) - F(v)| \geq \epsilon\}\right) \to 0 \text{ as } K \to 1,$$

where μ can denote both Lebesgue measure and μ_F.

Proof of Proposition 9: Recall that (h, j) denotes the public history in which bidder j was the last winner and won the object after history h. Let $V_i((h, j); \sigma)$ denote i's continuation value following history (h, j) under strategy profile σ.

Let $\rho \in \Delta(N \setminus i)$ be a probability distribution on the set of bidders excluding bidder i and define

$$\eta_i(h, \rho; \sigma) := \delta\left(V_i((h, i); \sigma)) - \sum_{j\ \in N\setminus i} \rho(j) V_i((h, j); \sigma)\right).$$

Define $\rho(j, h, \sigma, b_i)$ as the probability that j will win the object in the period following history h, given that all bidders other than i use the bid functions that are induced by their component of the profile σ, bidder i bids b_i and bidder i does not win the object. Then

$\rho(j, h, \sigma, b_i(\tilde{v}, , h, \sigma)) =$

$$\frac{1 - G_j(b_i(\tilde{v}, h, \sigma), h, \sigma)}{1 - \Pi_{l\neq i} G_l(b_i(\tilde{v}, h, \sigma), h, \sigma)} \int_{b_i(\tilde{v},h,\sigma)}^{\infty} \frac{\Pi_{l\neq i,j} G_l(b, h, \sigma)}{1 - G_j(b_i(\tilde{v}, h, \sigma), h, \sigma)} dG_j(b, h, \sigma).$$

Corollary 4 implies that for any $\epsilon^1 > 0$ we can find $\underline{K} < 1$ such that for all $K > \underline{K}$ and for any K-efficient bidding functions $b_i(\cdot, h, \sigma)$, $i = 1, \ldots, N$, there exist a set $\tilde{V} \subset [0, v^h]$ with measure (Lebesgue and μ_F) at least $1 - \epsilon^1$ such that for all $\tilde{v} \in \tilde{V} \cap [0, v^h - \epsilon^1]$:

$$\left|\rho(j, h, \sigma, b_i(\tilde{v}, h, \sigma)) - \frac{1}{1 - F^{N-1}(\tilde{v})} \int_{b_i([\tilde{v}, v^h], h, \sigma)} \Pi_{l\neq i,j} G_l(b, h, \sigma) dG_j(b, h, \sigma)\right| < \epsilon^1,$$

and by the rule for the change of variables for Lebesgue integrals

$$\left|\rho(j,h,\sigma,b_i(\tilde{v},h,\sigma)) - \frac{1}{1-F^{N-1}(\tilde{v})}\int_{\tilde{v}}^{\infty}\Pi_{l\neq i,j}G_l(b_l(v,h,\sigma),h,\sigma)dF(v)\right| < \epsilon^1.$$

Recall that if $|f_n| \leq g \in L^1$ and $f_n \to f$ in measure, then $\int f = \lim \int f_n$. Thus, using Corollary 4 again and the fact that $\Pi_{l\neq i,j}G_l(b,h,\sigma)$ is bounded by an L^1 function, we may infer that for any $\epsilon^1 > 0$ we can find $\underline{K} < 1$, if necessary larger than the one chosen before, such that for all $K > \underline{K}$ there exist a set $\tilde{V} \subset [0, v^h]$ with measure (Lebesgue and μ_F) at least $1-\epsilon^1$ such that for all $\tilde{v} \in \tilde{V} \cap [0, v^h - \epsilon^1]$:

$$\left|\rho(j,h,\sigma,b_i(\tilde{v},h,\sigma)) - \frac{1}{1-F^{N-1}(\tilde{v})}\int_{\tilde{v}}^{\infty} F^{N-2}(v)dF(v)\right| < \epsilon^1,$$

which finally implies that

$$\left|\rho(j,h,\sigma,b_i(\tilde{v},h,\sigma)) - \frac{1}{N-1}\right| < \epsilon^1$$

on $\tilde{V} \cap [0, v^h - \epsilon^1]$.

Let $v', v'' \in \tilde{V} \cap [\epsilon^2, v^h - \epsilon^2]$ for $\epsilon^2 > 0$ with $v' < v''$. Consider any $\hat{b}$ that satisfies $b_i(v',h,\sigma) \leq \hat{b} \leq b_i(v'',h,\sigma)$. The foregoing argument shows that we can choose v' and v'' arbitrarily close to each other provided we choose $\underline{K}$ sufficiently large. Note that

$$1 - \Pi_{l\neq i}G_l(b_i(v'',h,\sigma),h,\sigma) \leq 1 - \Pi_{l\neq i}G_l(\hat{b},h,\sigma) \leq 1 - \Pi_{l\neq i}G_l(b_i(v',h,\sigma),h,\sigma)$$

and

$$\begin{aligned}\int_{b_i(v',h,\sigma)}^{\infty}\Pi_{l\neq i,j}G_l(b,h,\sigma)dG_j(b,h,\sigma) &\leq \int_{\hat{b}}^{\infty}\Pi_{l\neq i,j}G_l(b,h,\sigma)dG_j(b,h,\sigma) \\ &\leq \int_{b_i(v'',h,\sigma)}^{\infty}\Pi_{l\neq i,j}G_l(b,h,\sigma)dG_j(b,h,\sigma).\end{aligned}$$

Therefore, for any $\epsilon^2 > 0$ we can choose $\epsilon^1 \in (0, \frac{\epsilon^2}{2})$ and $\underline{K}$ sufficiently large such that

$$\left|\rho(j,h,\hat{b}) - \frac{1}{N-1}\right| < \epsilon^1$$

for all $\hat{b} \in [b_i(\underline{v},h,\sigma), b_i(\overline{v},h,\sigma)]$ for some $\underline{v} \in \tilde{V} \cap [0, \epsilon^2)$ and $\overline{v} \in \tilde{V} \cap (v^h - \epsilon^2, v^h]$.

Let $B(\underline{v},\overline{v}) := [b_i(\underline{v},h,\sigma), b_i(\overline{v},h,\sigma)]$ and define

$$\eta_i^{\min} := \inf_{b_i \in B(\underline{v},\overline{v})} \eta_i(h, \rho(b_i,h,\sigma),\sigma)$$

and

$$\eta_i^{\max} := \sup_{b_i \in B(\underline{v}, \overline{v})} \eta_i(h, \rho(b_i, h, \sigma), \sigma).$$

Then, for any $\epsilon^3 > 0$, and for sufficiently large $\underline{K}$, we have

$$\eta_i^{\max} - \eta_i^{min} < \epsilon^3.$$

By Lemma 10 for any $\epsilon_1 > 0$ and $\epsilon_2 > 0$ we can choose $\underline{K}$ sufficiently large such that the allocation is (ϵ_1, ϵ_2)-monotonic. Let $\epsilon^4 > 6\epsilon_1$ such that the set $[v^h - \epsilon^2 - \epsilon^4, v^h - \epsilon^2]$ contains at least five consecutive elements $\varpi^l, \varpi^{l+1}, \ldots, \varpi^{l+4}$ of ϖ.

Let $v^l \in \varpi^l$, and $v^{l+2} \in \varpi^{l+2} \cap \tilde{V}$. For every $\epsilon^5 > 0$ we can choose $\underline{K} < 1$ sufficiently large such that

$$v^l + \eta_i^{\min} < b_i(v^{l+2}, h, \sigma) + \epsilon^5.$$

This can be seen as follows: Suppose instead that that there is $\epsilon^5 > 0$ such that for all all $\underline{K}$: $v^l + \eta_i^{\min} \geq b_i(v^{l+2}, h, \sigma) + \epsilon^5$. Use $b_{-i}^{(1)}$ to denote the highest bid, excluding i's bid, and consider the two mutually exclusive and exhaustive events $b_{-i}^{(1)} < b_i(v^l, h, \sigma)$ and $b_{-i}^{(1)} \geq b_i(v^l, h, \sigma)$. In the former case bidder i with valuation v^l suffers no loss from deviating to the bid $b_i(v^{l+2}, h, \sigma)$. In the latter case the expected change in payoffs from deviating is bounded from below by

$$\text{Prob}\left(b_{-i}^{(1)} < b_i(v^{l+2}, h, \sigma) | b_{-i}^{(1)} \geq b_i(v^l, h, \sigma)\right)\left[v^l + \eta_i^{\min} - b_i(v^{l+2}, h, \sigma)\right] - \left(1 - \text{Prob}\left(b_{-i}^{(1)} < b_i(v^{l+2}, h, \sigma) | b_{-i}^{(1)} \geq b_i(v^l, h, \sigma)\right)\right)\epsilon^3.$$

Note that $\text{Prob}\left(b_{-i}^{(1)} < b_i(v^{l+2}, h, \sigma) | b_{-i}^{(1)} \geq b_i(v^l, h, \sigma)\right)$ is bounded from below by the strictly positive probability of ϖ^{l+1}, that by assumption $\left[v^l + \eta_i^{\min} - b_i(v^{l+2}, h, \sigma)\right]$ is bounded from below by $\epsilon^5 > 0$ and that by choosing $\underline{K}$ sufficiently large, we can render ϵ^3 as close to zero as desired. This is in contradiction to σ being an equilibrium.

For any $\epsilon^6 > 0$ we also have

$$b_i(v^{l+2}, h, \sigma) < \epsilon^6$$

provided we choose $\underline{K}$ large enough. To see this, note that the probability that the highest valuation is in ϖ^{l+4} while the second highest valuation is in ϖ^{l+3} is bounded away from zero for sufficiently high $\underline{K}$. Since in that case the price paid exceeds $b_i(v^{l+2}, h, \sigma)$, the expected price would be bounded away from zero if the condition were not met.

Combining what we have learned we get,

$$\eta_i^{\min} < \epsilon^5 + \epsilon^6 - v^l,$$

which implies that

$$\eta_i^{\max} < -v^l + \epsilon^3 + \epsilon^5 + \epsilon^6 < -v^h + \epsilon^2 + \epsilon^3 + \epsilon^4 + \epsilon^5 + \epsilon^6,$$

since $v^l \in (v^h - \epsilon^4 - \epsilon^2, v^h - \epsilon^2]$. Therefore, for any $v_i \in [\epsilon^2, v^h - (\epsilon^2 + \epsilon^3 + \epsilon^4 + \epsilon^5 + \epsilon^6)]$ bidder i strictly prefers not to submit a bid, which for sufficiently small ϵ^j's contradicts K-efficiency for sufficiently large $\underline{K}$. □

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